ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

Bresets

Lokavarapu Sujatha, Nistala V.E.S. Murthy

Ph.D. Student, Department of Mathematics, AUCST, Andhra University, Andhra Pradesh, India

Professor, Department of Mathematics, AUCST, Andhra University, Andhra Pradesh, India

ABSTRACT*:* Several objects like poset, (complete) semi-lattice, (complete) lattice, graph etc., have the underlying object, set with a binary relation. In this paper these sets with binary relations called bresets are exclusively studied together with their substructures and their lattice algebraic properties.

KEYWORDS*:* Binary relations, (Infinite) digraphs, (l-sub, u-sub, sub) bresets.

I. INTRODUCTION

Binary relations are one of the earliest relations known to both mankind and Mathematicians. Basic relations like reflexive, symmetric, transitive, irreflexive, anti-symmetric, cyclic, etc. are all binary relations which play an important role in the studies of several order-structures like semi ordered set, well ordered set, totally ordered set, partial ordered set and higher-order structures like, join (meet) semi lattice, (distributive, modular, deMorgan) lattice, (join,meet) and still higher order structures like complete lattice, infinite (join,meet) distributive lattice, completely distributive complete lattice etc.. Notice that the under lying object for all of them is a set with a binary relation. For more information in the studies of these higher ordered objects, one can refer to Szasz[19], Birkhoff[4] etc..

Another important class of objects, on a set with a binary relation, is the graph which is nothing but a finite set (of nodes) together with a binary relation (of edges). The Theory of Graphs is well known for its applications both in Hardware and Software of Computer Science. In Hardware, it is used in the feasibility, design and analysis of Circuits. For more details in this direction, one can refer to Charles Desoer and Ernest Kuh[5], Krishnaiyan Thulasiraman[12] and Narasingh Deo[14]. In Software, there are several applications for both notions and algorithms of Graph Theory. To name a few, shortest path and minimal spanning tree are chiefly studied in communication/transportation networks; various types of connectedness, cycles etc. are used in Digital Image Processing/Detection; BFS (Breadth First Search), DFS (Depth First Search) are extensively used in Searching. For more information in this direction one can refer to Kenneth H Rosen[11], J.P. Trembly and R. Manohar[19] and Joe L. Mott, Abraham Kandel and Theodore P. Baker[9]. Interestingly, there are even computer languages like HINT (an extension of LISP), GRASPE (another extension of LISP), GEA (Graphic Extended ALGOL, an extension of ALGOL), GIRL (Graph Information Retrieval Language), GTPL (Graph Theoretic Processing Language) etc. and also packages like GASP (Graph Algorithms Software Package), SPANTREE (To find a spanning tree in the given graph) to exclusively process Graph Theoretic ideas/algorithms.
However, a set with a binary operation, as an object by itself is not exclusively studied and primarily we take up this aspect in this thesis. We call a set with a binary relation a breset (Binarily RElated SET) and make an exclusive study of this object. Since a digraph is a finite set with a (finite) binary relation and since the notion of a breset has no restriction of finiteness either on the number of elements of the underlying set or on the size of the binary relation, a breset can be regarded as an infinite digraph and hence a generalization of digraph. Of course, infinite (di) graphs were studied in conjunction with groups and/or vector spaces. To see some work in this direction, one can refer to, Finucane[7], Seiffer[17], Soardi and Woess[18], Bondy and Hemminger[3], Andrea[2] etc..

In the process, we extend such notions of (di) graphs as conjunctive product (also known as tensor or categorical product), disjunctive product (also known as co-normal product) to *arbitrary* families of bresets and study them. Also we introduce and study such notions as factors, radicals for bresets and prove such results as: for a family of bresets $(\mathsf{A}_i)_{i\in I}$, (a) A_i is reflexive (symmetric, transitive, anti-symmetric) for all $i \in I$ if and only if $\prod^{c}_{i\in I}\mathsf{A}_i$ is reflexive (symmetric, transitive, anti-symmetric) when ever each A_i is non empty for all $i \in I$ under some simple different conditions and (lattice) algebraic properties of (inverse) images of substructures of bresets under (function and relation)

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

morphisms between bresets etc. all of which are more of purely mathematical interest. It is for these reasons, in fact, we preferred the word breset over the word infinite digraph.

However, since some of the new notions, in the finite set up, are relevant for computer scientists as well and since $(a_1,a_2)\leftrightarrow a_1a_2$ defines one-one correspondence between the product set $A_1\times A_2$ and the string set A_1A_2, where $A_1\times A_2 = \{(a_1,a_2)\,|\,a_i\in A_i, i=1,2,..\}$ and $A_1A_2 = \{a_1a_2\,|\,a_i\in A_i, i=1,2,..\}$, which is easily extendable to n-tuples and n-strings(strings of length n), in general, we do not distinguish between n-tuples and n-strings and in fact, we prefer to use n-strings in stead of n-tuples especially in our examples and counter examples.
Note: Whenever a notion that we used and/or introduced for bresets is already known in a (finite, infinite) (di) graph theory and we are aware, in almost all cases we make an explicit mention of the same and explain the relation between them.

II. BRESETS AND SUBSTRUCTURES OF BRESETS

In this section the notions of breset, lower sub breset, upper sub breset and sub breset are introduced and some relations between them are studied along with some examples and counter examples.

Definitions and Statements 2.1 *(a) A breset is any ordered pair (A,R), where A is called the underlying set or shortly the u-set of (A,R) and R is a binary relation on A.*

Let us recall that a binary relation R on a set A is any subset of $A\times A$.

(b) For any pair of bresets $(A,R),(B,S)$, (A,R) is *equal* to (B,S), denoted by $(A,R) = (B,S)$, if and only if $A = B$ and $R = S$.

(c) A breset (A,R) is *empty breset* or simply *empty*, denoted by Φ, iff the underlying set $A = \phi$ and the binary relation $R = \phi$.

(d) Clearly, a breset (A,R) is empty if and only if $A=\phi$. Further, (a) in a breset (A,R),it can so happen that the u-set A is nonempty but the binary relation R on A is empty and (b) there can be several bresets with the same u-set A.

Notation: Since a breset (A,R) is uniquely determined by its binary relation R on the set A, here onwards for notation convenience, we denote the breset (A,R) by A and the binary relation R by β_{A}. Also whenever, *further, through out this and other chapters on bresets, the script letters and the suffixed script letters always stand for the bresets. In other words, the* P *stands for the ordered pair* (P,β_{P}), *etc..*

Definitions 2.2 *Let* A *be a breset.*

(a) A is said to be reflexive, irreflexive, symmetric, asymmetric, anti-symmetric, transitive if and only if β_{A} is so on A.

(b) A is said to be equivalence (quasi-equivalence, partial order, pre-order/ quasi-order, tolerance/compatibility) if and only if β_{A} is so on A.

(c) A is said to be (meet, join) (complete) semi lattice if and only if β_{A} is so on A.

(d) A is said to be (distributive, modular)(complete) lattice if and only if β_{A} is so on A.

(e) A is said to be infinite (meet, join) distributive lattice if and only if β_{A} is so on A.

Definitions and Statements 2.3 *Let* A, B *be a pair of bresets.*

a. A is said to be a *sub system* of B iff $A\subseteq B$.

b. A is said to be a *lower sub breset* or simply l-sub breset of B iff $A\subseteq B$ and $\beta_{\mathsf{A}}\subseteq\beta_{\mathsf{B}}\cap(A\times A)$.

c. For any breset X, the set of all l-sub bresets of X is denoted by $\mathsf{S}_l(\mathsf{X})$.

d. A is said to be a *upper sub breset* or simply u-sub breset of B iff $A\subseteq B$ and $\beta_{\mathsf{A}}\supseteq\beta_{\mathsf{B}}\cap(A\times A)$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 1, Issue 5 , December 2014

e. For any breset X, the set of all upper sub bresets of X is denoted by $\mathsf{S}_u(\mathsf{X})$.

f. A is said to be a *sub breset* of B iff $A \subseteq B$ and $\beta_{\mathsf{A}} = \beta_{\mathsf{B}} \cap (A \times A)$.

g. For any breset X, the set of all sub bresets of X is denoted by $\mathsf{S}(\mathsf{X})$.

Remark: Clearly, when the underlying set A of the breset A is finite, the notion of l-sub breset of a breset is equivalent to the notion of sub digraph of a digraph and the notion of sub breset of a breset is equivalent to the notion of induced sub digraph.

Lemma 2.4 *For any breset* X

(1) Being l-sub breset is a binary relation on $\mathsf{S}_l(\mathsf{X})$ making it a breset and further, a poset.

(2) Being u-sub breset is a binary relation on $\mathsf{S}_u(\mathsf{X})$ making it a breset and further, a poset.

(3) Being sub breset is a binary relation on $\mathsf{S}(\mathsf{X})$ making it a breset and further, a poset.

Proof: (1): (a) For any A in $\mathsf{S}_l(\mathsf{X})$, since $A \subseteq A$, $\beta_{\mathsf{A}} \subseteq \beta_{\mathsf{A}} \cap (A \times A) = \beta_{\mathsf{A}}$, by 2.3(b), A is a l-sub breset of A so that being l-sub breset is reflexive.

(b) Let A be an l-sub breset of B and B be an l-sub breset of A. Then $A \subseteq B$, $\beta_{\mathsf{A}} \subseteq \beta_{\mathsf{B}} \cap (A \times A)$, $B \subseteq A$ and $\beta_{\mathsf{B}} \subseteq \beta_{\mathsf{A}} \cap (B \times B)$ from which follow $A = B$ and $\beta_{\mathsf{A}} = \beta_{\mathsf{B}}$. Hence, $\mathsf{A} = \mathsf{B}$ and being l-sub breset is anti-symmetric.

(c) Let A be an l-sub breset of B and B be an l-sub breset of C. Then $A \subseteq B$, $B \subseteq C$, $\beta_{\mathsf{A}} \subseteq \beta_{\mathsf{B}} \cap (A \times A)$ and $\beta_{\mathsf{B}} \subseteq \beta_{\mathsf{C}} \cap (B \times B)$ from which follow $A \subseteq C$, $\beta_{\mathsf{A}} \subseteq \beta_{\mathsf{C}} \cap (A \times A)$. Now by 2.3(b), A is an l-sub breset of C and being an l-sub breset is transitive.

From (a),(b) and (c), it follows that being l-sub breset is a partial order, making $\mathsf{S}_l(\mathsf{X})$ a poset.

(2): (a) For any A in $\mathsf{S}_u(\mathsf{X})$, since $A \subseteq A$, $\beta_{\mathsf{A}} \supseteq \beta_{\mathsf{A}} \cap (A \times A) = \beta_{\mathsf{A}}$, by 2.3(d), A is a u-sub breset of A so that being u-sub breset is reflexive.

(b) Let A be a u-subbreset of B and B be a u-sub breset of A. Then $A \subseteq B$, $\beta_{\mathsf{A}} \supseteq \beta_{\mathsf{B}} \cap (A \times A)$, $B \subseteq A$ and $\beta_{\mathsf{B}} \supseteq \beta_{\mathsf{A}} \cap (B \times B)$ from which follow $A = B$ and $\beta_{\mathsf{A}} = \beta_{\mathsf{B}}$. Hence, $\mathsf{A} = \mathsf{B}$ and being u-sub breset is anti-symmetric.

(c) Let A be a u-sub breset of B and B be a u-sub breset of C. Then $A \subseteq B$, $B \subseteq C$, $\beta_{\mathsf{A}} \supseteq \beta_{\mathsf{B}} \cap (A \times A)$ and $\beta_{\mathsf{B}} \supseteq \beta_{\mathsf{C}} \cap (B \times B)$ from which follow $A \subseteq C$, $\beta_{\mathsf{A}} \supseteq \beta_{\mathsf{C}} \cap (A \times A)$. Now by 2.3(d), A is a u-sub breset of C and being u-sub breset is transitive.

From (a),(b) and (c), it follows that being u-sub breset is a partial order, making $\mathsf{S}_u(\mathsf{X})$ a poset.

(3): (a) For any A in $\mathsf{S}(\mathsf{X})$, since $A \subseteq A$, $\beta_{\mathsf{A}} = \beta_{\mathsf{A}} \cap (A \times A) = \beta_{\mathsf{A}}$, by 2.3(f), A is a sub breset of A so that being sub breset is reflexive.

(b) Let A be a sub breset of B and B be a sub breset of A. Then $A \subseteq B$, $\beta_{\mathsf{A}} = \beta_{\mathsf{B}} \cap (A \times A)$, $B \subseteq A$ and $\beta_{\mathsf{B}} = \beta_{\mathsf{A}} \cap (B \times B)$ from which follow $A = B$ and $\beta_{\mathsf{A}} = \beta_{\mathsf{B}}$. Hence, $\mathsf{A} = \mathsf{B}$ and being sub breset is anti-symmetric.

(c) Let A be a sub breset of B and B be a sub breset of C. Then $A \subseteq B$, $B \subseteq C$, $\beta_{\mathsf{A}} = \beta_{\mathsf{B}} \cap (A \times A)$ and $\beta_{B} = \beta_{\mathsf{C}} \cap (B \times B)$ from which follow $A \subseteq C$, $\beta_{\mathsf{A}} = \beta_{\mathsf{B}} \cap (A \times A) = \beta_{\mathsf{C}} \cap (B \times B) \cap (A \times A) = \beta_{\mathsf{C}} \cap ((A \cap B) \times (A \cap B)) = \beta_{\mathsf{C}} \cap (A \times A)$. Now by 2.3(f), A is a sub breset of C and being sub breset is transitive.

From (a),(b) and (c), it follows that being sub breset is a partial order, making $\mathsf{S}(\mathsf{X})$ a poset.

Remarks: (1) Clearly, every sub breset is a lower (upper) sub breset.
(2) An l-sub breset need *not* be a sub breset as shown in example 2.5 below.
(3) A u-sub breset need *not* a sub breset as shown in example 2.6 below.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 1, Issue 5 , December 2014

(4) For any breset B and for any subset A of B, the subset $\beta_{\mathsf{A}} = \beta_{\mathsf{B}} \cap (A\times A)$ of $B\times B$, is such that A is always a sub breset of B, called the induced sub breset.

Example 2.5 $A=\{1,2,3\}$, $B=\{1,2,3,4\}$, $\beta_{\mathsf{A}}=\{(1,2),(1,3)\}$ and $\beta_{\mathsf{B}}=\{(1,1),(1,2),(1,3),(1,4)\}$. Then $A\times A=\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ and $\beta_{\mathsf{B}}\cap(A\times A)$ = $\{(1,1),(1,2),(1,3),(1,4)\}\cap\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ = $\{(1,1),(1,2),(1,3)\}$. Then clearly, $\beta_{\mathsf{A}}\subseteq\beta_{\mathsf{B}}\cap(A\times A)$ and $\beta_{\mathsf{A}}\neq\beta_{\mathsf{B}}\cap(A\times A)$, implying that A is an l-sub breset of B and *not* a sub breset of B.

Example 2.6 $A=\{1,2,3\}$, $B=\{1,2,3,4\}$, $\beta_{\mathsf{A}}=\{(1,2),(1,3),(2,3)\}$ and $\beta_{\mathsf{B}}=\{(1,2),(1,3),(1,4)\}$. Then $A\times A$ = $\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ and $\beta_{\mathsf{B}}\cap(A\times A)$ = $\{(1,2),(1,3),(1,4)\}\cap\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ = $\{(1,2),(1,3)\}$. Then clearly, $\beta_{\mathsf{A}}\supseteq\beta_{\mathsf{B}}\cap(A\times A)$ and $\beta_{\mathsf{A}}\neq\beta_{\mathsf{B}}\cap(A\times A)$. So, A is a u-sub breset of B and *not* a sub breset of B.

Example 2.7 $A=\{1,2,3\}$, $B=\{1,2,3,4\}$, $\beta_{\mathsf{A}}=\{(1,2),(1,3),(2,3)\}$ and $\beta_{\mathsf{B}}=\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$. Then $A\times A$ = $\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ and $\beta_{\mathsf{B}}\cap(A\times A)$ = $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}\cap\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$ = $\{(1,2),(1,3),(2,3)\}$. Then clearly, $\beta_{\mathsf{A}} = \beta_{\mathsf{B}}\cap(A\times A)$. So, A is a sub breset of B.

III. LATTICE ALGEBRAIC PROPERTIES OF SUB STRUCTURES OF BRESETS

In this section first we introduce the notions of union and intersection for bresets which essentially generalize the existing notions of union (Cf. P28, Jorgen-Gregory[10]) and intersection (Cf. P195, Jorgen-Gregory[10]) for digraphs and next we use these notions to investigate order structures on collections of substructures of bresets.

Lemma 3.1 *For any family of bresets* $(\mathsf{A}_i)_{i\in I}$, A *where* $A = \cap_{i\in I} A_i$, $\beta_{\mathsf{A}} = \cap_{i\in I}\beta_{\mathsf{A}_i}$, *is a breset.*

Proof: Since, $(\mathsf{A}_i)_{i\in I}$ is a family of bresets, $\beta_{\mathsf{A}_i}\subseteq A_i\times A_i$ for all $i\in I$. Since $\cap_{i\in I}(A_i\times A_i)$ = $\cap_{i\in I} A_i\times\cap_{i\in I} A_i$, $\beta_{\mathsf{A}} = \cap_{i\in I}\beta_{\mathsf{A}_i}\subseteq\cap_{i\in I}(A_i\times A_i)$ = $\cap_{i\in I} A_i\times\cap_{i\in I} A_i$ = $A\times A$ and A becomes a breset with this binary relation.

Definition 3.2 *For any family of bresets* $(\mathsf{A}_i)_{i\in I}$, *the breset* A *defined as in 3.1 above is called the intersection of bresets* $(\mathsf{A}_i)_{i\in I}$ *and is denoted by* $\cap_{i\in I}\mathsf{A}_i$.

In other words, for bresets $(\mathsf{A}_i)_{i\in I}$, (a) $\cap_{i\in I}\mathsf{A}_i = (\cap_{i\in I} A_i, \cap_{i\in I}\beta_{\mathsf{A}_i})$ (b) $\beta_{\cap_{i\in I}\mathsf{A}_i} = \cap_{i\in I}\beta_{\mathsf{A}_i}$.

Remark: Notice that the notion of intersection for both graphs and digraphs is available as follows: Refer page 177, Jorgen-Gregory[10] for finite intersection of digraphs with the same vertex set and refer page 3, Diestel[5] for intersection of graphs.

Lemma 3.3 *For any family of bresets* $(\mathsf{A}_i)_{i\in I}$, A *where* $A = \cup_{i\in I} A_i$, $\beta_{\mathsf{A}} = \cup_{i\in I}\beta_{\mathsf{A}_i}$, *is a breset.*

Proof: Since, $(\mathsf{A}_i)_{i\in I}$ is a family of bresets, $\beta_{\mathsf{A}_i}\subseteq A_i\times A_i$ for all $i\in I$. Since $\cup_{i\in I}(A_i\times A_i)\subseteq\cup_{i\in I} A_i\times\cup_{i\in I} A_i$, $\beta_{\mathsf{A}} = \cup_{i\in I}\beta_{\mathsf{A}_i}\subseteq\cup_{i\in I}(A_i\times A_i)$ $\subseteq$ $\cup_{i\in I} A_i\times\cup_{i\in I} A_i$ = $A\times A$ and A becomes a breset with this binary relation.

Definition 3.4 *For any family of bresets* $(\mathsf{A}_i)_{i\in I}$, *the breset* A *defined as in 3.3 above is called the union of bresets* $(\mathsf{A}_i)_{i\in I}$ *and is denoted by* $\cup_{i\in I}\mathsf{A}_i$.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 1, Issue 5 , December 2014

In other words, for bresets $(\mathsf{A}_i)_{i\in I}$, (a) $\cup_{i\in I}\mathsf{A}_i = (\cup_{i\in I} A_i, \cup_{i\in I}\beta_{\mathsf{A}_i})$ (b) $\beta_{\cup_{i\in I}\mathsf{A}_i} = \cup_{i\in I}\beta_{\mathsf{A}_i}$.

Remarks: (1) Notice that (a) the notion of union is already available for (pseudo) (di) graphs etc. and it is *not* unique. For example, refer page 10, Jorgen-Gregory[10] for union of pseudo digraphs, refer page 3, Diestel[6] for union of graphs (b) the union (di) graphs can become a pseudo (di) graph and (c) although when the underlying set is finite, the notions of digraph and breset are exactly the same, when one takes their union for a pair of digraphs, one may end up getting a pseudo digraph in which more than two edges can exist between a pair of nodes, which will not happen in our union because the set union does not allow multiple entries for a same set element (as in a multiset).
(2) Although we could define arbitrary union and arbitrary intersection for even class-indexed families of bresets in the same way as we defined above, we *do not* take such collection of bresets for fear of unions of such collections of bresets can easily become a non-breset and in fact could make a pair of classes (cf. any book on Category Theory, for example, Herrlich-Strecker[8]). However, an application of the union and the intersection of a set-indexed family of bresets to (lower, upper) sub bresets of a given breset will be useful as can be seen in the following results.

Lemma 3.5 *For any breset* X *and for any family of l-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *of* X, *the set* $A = \cap_{i\in I} A_i$ *together with the binary relation* $\beta_{\mathsf{A}} = \cap_{i\in I}\beta_{\mathsf{A}_i}$ *is a subsystem* A *of* X *such that*

(1) $\mathsf{A} = \cap_{i\in I}\mathsf{A}_i$ is an l-sub breset of X

(2) A is an l-sub breset of A_i for all $i\in I$

(3) whenever B is an l-sub breset of A_i for all $i\in I$, B is an l-sub breset of A.

Proof: (1): Clearly, by 3.1 and 3.2, the equality follows and $\mathsf{A} = \cap_{i\in I}\mathsf{A}_i$ is a breset. Since $A\subseteq X$ and $\beta_{\mathsf{A}}\subseteq\beta_{\mathsf{A}_i}\cap(A\times A)\subseteq\beta_{\mathsf{X}}\cap(A\times A)$, A is an l-sub breset of X.

(2): A is an l-sub breset of A_i for all $i\in I$ because (a) $A\subseteq A_i$ and (b) $\beta_{\mathsf{A}} = \cap_{i\in I}\beta_{\mathsf{A}_i}\subseteq\beta_{\mathsf{A}_i}$ and already $\beta_{\mathsf{A}}\subseteq A\times A$, so $\beta_{\mathsf{A}}\subseteq\beta_{\mathsf{A}_i}\cap(A\times A)$.

(3): Let B be an l-sub breset of A_i for all $i\in I$. Then $B\subseteq A_i$ and $\beta_{\mathsf{B}}\subseteq\beta_{\mathsf{A}_i}\cap(B\times B)\subseteq\beta_{\mathsf{A}_i}$, $B\times B$ which imply (a) $B\subseteq\cap_{i\in I}A_i = A$ and (b) $\beta_{\mathsf{B}}\subseteq\beta_{\mathsf{A}_i}$ for all $i\in I$, and always $\beta_{\mathsf{B}}\subseteq B\times B$. So, $\beta_{\mathsf{B}}\subseteq\beta_{\mathsf{A}}\cap(B\times B)$.
From (a) and (b) we get that B is an l-sub breset of A.

Definition 3.6 *For any breset* X *and for any family of l-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *of* X, *the l-sub breset* A *defined as in the above Lemma 3.5 is called the intersection of l-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *and is denoted by* $\cap_{i\in I}\mathsf{A}_i$.

Lemma 3.7 *For any breset* X *and for any family of l-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *of* X, *the set* $A = \cup_{i\in I} A_i$ *together with the binary relation* $\beta_{\mathsf{A}} = \cup_{i\in I}\beta_{\mathsf{A}_i}$ *is a subsystem* A *of* X *such that*

(1) $\mathsf{A} = \cup_{i\in I}\mathsf{A}_i$ is an l-sub breset of X

(2) each A_i is an l-sub breset of A

(3) whenever A_i is an l-sub breset of B for all $i\in I$, A is an l-sub breset of B.

Proof: (1): Clearly, by 3.3 and 3.4, the equality follows and $\mathsf{A} = \cup_{i\in I}\mathsf{A}_i$ is a breset. Since $A\subseteq X$ and $\beta_{\mathsf{A}}\subseteq(\cup_{i\in I}\beta_{\mathsf{A}_i})\cap(A\times A)\subseteq\beta_{\mathsf{X}}\cap(A\times A)$, A is an l-sub breset of X.

(1): A_i is an l-sub breset of A because (a) $A_i\subseteq\cup_{i\in I}A_i = A$ and (b) always $\beta_{\mathsf{A}_i}\subseteq A_i\times A_i$ and $\beta_{\mathsf{A}_i}\subseteq\cup_{i\in I}\beta_{\mathsf{A}_i} = \beta_{\mathsf{A}}$, so that $\beta_{\mathsf{A}_i}\subseteq\beta_{\mathsf{A}}\cap(A_i\times A_i)$.

(2) Let A_i be an l-sub breset of B for all $i\in I$. Then $A_i\subseteq B$ and $\beta_{\mathsf{A}_i}\subseteq\beta_{\mathsf{B}}\cap(A_i\times A_i)\subseteq\beta_{\mathsf{B}}, A_i\times A_i$.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

Clearly, (a) $A = \cup_{i\in I} A_i \subseteq B$ and (b) $\beta_{\mathsf{A}} = \cup_{i\in I} \beta_{\mathsf{A}_i} \subseteq \beta_{\mathsf{B}}$ and already $\beta_{\mathsf{A}} \subseteq A\times A$ implying that $\beta_{\mathsf{A}} \subseteq \beta_{\mathsf{B}} \cap (A\times A)$. From (a) and (b) we get that A is an l-sub breset of B.

Definition 3.8 *For any breset* X *and for any family of l-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *of* X, *the l-sub breset* A *defined as in the above Lemma 3.7 is called the union of l-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *and is denoted by* $\cup_{i\in I} \mathsf{A}_i$.

Theorem 3.9 *The set of all l-sub bresets of a breset is a complete lattice.*

Proof: Let X be a breset and let $\mathsf{S}_l(\mathsf{X})$ be the set of all l-sub bresets of X. Then $\mathsf{S} = \mathsf{S}_l(\mathsf{X}) = \{\mathsf{A} \mid \mathsf{A}$ is an l-sub breset of $\mathsf{X}\}$.

For any pair of elements $\mathsf{A},\mathsf{B} \in \mathsf{S}$, let $A \le B$ iff A is an l-sub breset of B or equivalently, $A \subseteq B$ and $\beta_{\mathsf{A}} \subseteq \beta_{\mathsf{B}} \cap (A\times A)$. Then $\le$ defines a partial order on S as seen in 2.4(1).

Let $(\mathsf{A}_i)_{i\in I}$ be a non empty family of l-sub bresets of X. Then from the Lemma 3.5, $\mathsf{A} = \cap_{i\in I} \mathsf{A}_i$ is a breset such that (1) it is an l-sub breset of both A_i and X, and (2) whenever B is an l-subbreset of X such that B is also an l-sub breset of A_i, then B is an l-subbreset of $\cap_{i\in I} \mathsf{A}_i$, implying that $\mathsf{A} = \cap_{i\in I} \mathsf{A}_i$ is the greatest lower bound of $(\mathsf{A}_i)_{i\in I}$ in S.

Also, from the Lemma 3.7, $\mathsf{A} = \cup_{i\in I} \mathsf{A}_i$ is a breset such that (1) A_i is an l-sub breset of A (2) whenever B is an l-sub breset of X such that A_i is an l-sub breset of B, implying that $\cup_{i\in I} \mathsf{A}_i$ is the least upper bound of $(\mathsf{A}_i)_{i\in I}$ in S.

Thus, S is a complete lattice.

Lemma 3.10 *For any breset* X *and for any family of u-sub bresets* $(\mathsf{A}_i)_{i\in I}$ *of* X, *the set* $A = \cap_{i\in I} A_i$ *together with the binary relation* $\beta_{\mathsf{A}} = \cap_{i\in I} \beta_{\mathsf{A}_i}$ *is a subsystem* A *of* X *such that*

(1) $\mathsf{A} = \cap_{i\in I} \mathsf{A}_i$ is a u-sub breset of X but not necessarily of A_i

(2) whenever B is a u-sub breset of A_i for all $i \in I$, then B is a u-sub breset of A.

Proof: (1): Clearly, by 3.1 and 3.2, the equality follows and A is a breset. Since A_i is a u-sub breset of X, $A_i \subseteq X$ and $\beta_{\mathsf{A}_i} \supseteq \beta_{\mathsf{X}} \cap (A_i \times A_i)$.

So, (a) $A = \cap_{i\in I} A_i \subseteq A_i \subseteq X$ and (b) $\cap_{i\in I} \beta_{\mathsf{A}_i} \supseteq \cap_{i\in I} (\beta_{\mathsf{X}} \cap (A_i \times A_i)) = \beta_{\mathsf{X}} \cap (\cap_{i\in I} (A_i \times A_i)) = \beta_{\mathsf{X}} \cap (\cap_{i\in I} A_i \times \cap_{i\in I} A_i) = \beta_{\mathsf{X}} \cap (A\times A)$.

From (a) and (b), we get that A is a u-sub breset of X.

(2) Let B be a u-sub breset of A_i for all $i \in I$. Then $B \subseteq A_i$ and $\beta_{\mathsf{B}} \supseteq \beta_{\mathsf{A}_i} \cap (B\times B)$ for all $i \in I$.

So, (a) $B \subseteq \cap_{i\in I} A_i = A$ and (b) $\beta_{\mathsf{B}} \supseteq \beta_{\mathsf{A}_i} \cap (B\times B) \supseteq \cap_{i\in I} \beta_{\mathsf{A}_i} \cap (B\times B) = \beta_{\mathsf{A}} \cap (B\times B)$.

From (a) and (b), we get that B is a u-sub breset of A.

Example 3.11 $A_1 = \{a,b,c\}$, $A_2 = \{b,c,d\}$, $\beta_{\mathsf{A}_1} = \{ab,ac\}$, $\beta_{\mathsf{A}_2} = \{bc,bd\}$. *Then* $A = A_1 \cap A_2 = \{b,c\}$, $\beta_{\mathsf{A}} = \beta_{\mathsf{A}_1} \cap \beta_{\mathsf{A}_2} = \phi \supset \beta_{\mathsf{A}_2} \cap (A\times A) = \{bc,bd\} \cap \{bb,bc,cb,cc\} = \{bc\}$ *implies* $\mathsf{A} = (A_1 \cap A_2, \beta_{\mathsf{A}_1} \cap \beta_{\mathsf{A}_2})$ *is not a u-sub breset of* A_2.

Remarks: In what follows we show that an analogous Lemma with l-sub breset replaced by u-sub breset in 3.7 is *no* longer true. In other words, for any breset X and for any family of u-sub bresets $(\mathsf{A}_i)_{i\in I}$ of X, the set $A = \cup_{i\in I} A_i$ together with the binary relation $\beta_{\mathsf{A}} = \cup_{i\in I} \beta_{\mathsf{A}_i}$ is a breset. However

(1) $\mathsf{A} = \cup_{i\in I} \mathsf{A}_i$ is *not* necessarily a u-sub breset of X.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

(2) A_i is *not* necessarily a u-sub breset of $\mathsf{A} = \cup_{i\in I} \mathsf{A}_i$.

(3) whenever B is a u-sub breset of X such that A_i is a u-sub breset of B then A is *not* necessarily a u-sub breset of B

as can be seen in the following examples:

Example 3.12 *Let* $A = \{a\}$, $B = \{b\}$, $A^2 = \{aa\}$, $B^2 = \{bb\}$, $X = \{a,b\}$, $\beta_{\mathsf{X}} = \{ab\}$, $\beta_{\mathsf{A}} = \{aa\}$, $\beta_{\mathsf{B}} = \{bb\}$, $\beta_{\mathsf{A\cup B}} = \{aa,bb\}$, $(A\cup B)^2 = \{aa,ab,ba,bb\}$, $\beta_{\mathsf{X}} \cap (A\cup B)^2 = \{ab\}$. *Then, (1)* $\beta_{\mathsf{A}} \supseteq A^2 \cap \beta_{\mathsf{X}}$ *implying that* A *is a u-sub breset of* X *(2)* $\beta_{\mathsf{B}} \supseteq B^2 \cap \beta_{\mathsf{X}}$ *implying that* B *is a u-sub breset of* X *(3)* $\beta_{\mathsf{X}} \cap (A\cup B)^2 \not\supseteq \beta_{\mathsf{A\cup B}}$, *implying that* $\mathsf{A}\cup\mathsf{B}$ *is not a u-sub breset of* X *over through* A *and* B *are u-sub bresets of* X.

Example 3.13 *Let* $A = \{a\}$, $B = \{a,b\}, A\cup B = \{a,b\}$, $\beta_{\mathsf{A}} = \{\phi\}$, $\beta_{\mathsf{B}} = \{aa\}$, $\beta_{\mathsf{A\cup B}} = \{aa\}$. *Then* $\mathsf{A}, \mathsf{A}\cup\mathsf{B}$ *are bresets such that* A *is not a u-sub breset* $\mathsf{A}\cup\mathsf{B}$, *because* $\phi = \beta_{\mathsf{A}} \not\supseteq A^2 \cap \beta_{\mathsf{A\cup B}} = \{aa\}$. *Therefore,* A *need not be a u-sub breset of* $\mathsf{A}\cup\mathsf{B}$.

Example 3.14 *Let* $A = \{a\}$, $B = \{b,c\}$, $C = \{a,b,c\}$, $X = \{a,b,c\}$, $\beta_{\mathsf{A}} = \{aa\}$, $\beta_{\mathsf{B}} = \{cc\}$, $\beta_{\mathsf{C}} = \{ab,ba,cc\}$, $\beta_{\mathsf{X}} = \{ab,ba\}$. *Then*

(1) $\beta_{\mathsf{A}} \supseteq A^2 \cap \beta_{\mathsf{C}} = \phi$. So, A is a u-sub breset of C.

(2) $\beta_{\mathsf{B}} = B^2 \cap \beta_{\mathsf{C}} = \{cc\}$. So, B is a u-sub breset of C.

(3) $\{ab,ba,cc\} = \beta_{\mathsf{C}} \supset C^2 \cap \beta_{\mathsf{X}} = \{ab,ba\}$. So, C is a u-sub breset of X.

(4) $\{aa,cc\} = \beta_{\mathsf{A\cup B}} \not\supseteq (A\cup B)^2 \cap \beta_{\mathsf{C}} = \{ab,ba,cc\}$. So, $\mathsf{A}\cup\mathsf{B}$ is *not* a u-sub breset of C.

(5) A is u-sub breset of C, B is a u-sub breset of C but $\mathsf{A}\cup\mathsf{B}$ is *not* a u-sub breset of C.

Lemma 3.15 *For any breset* X *and for any family of sub bresets* $(\mathsf{A}_i)_{i\in I}$ *of* X, *the set* $A = \cap_{i\in I} A_i$ *together with the binary relation* $\beta_{\mathsf{A}} = \cap_{i\in I} \beta_{\mathsf{A}_i}$ *is a subsystem of* X *such that*

(1) $\cap_{i\in I} \mathsf{A}_i$ is a sub breset of X

(2) $\cap_{i\in I} \mathsf{A}_i$ is a sub breset of A_i for all $i \in I$

(3) whenever B is a sub breset of A_i for all $i \in I$, B is a sub breset of A.

Proof: (1): Since A_i is a sub breset of X, $\beta_{\mathsf{A}_i} = A_i^2 \cap \beta_{\mathsf{X}}$. Now $\beta_{\cap_{i\in I}\mathsf{A}_i} = \cap_{i\in I} \beta_{\mathsf{A}_i} = \cap_{i\in I} (A_i^2 \cap \beta_{\mathsf{X}}) = (\cap_{i\in I})^2 \cap \beta_{\mathsf{X}}$ which implies $\cap_{i\in I} \mathsf{A}_i$ is a sub breset of X.

(2): To show $\cap_{i\in I} \mathsf{A}_i$ is a sub breset of A_{i_0}, it is enough to show that $\beta_{\cap_{i\in I}\mathsf{A}_i} = \cap_{i\in I} \beta_{\mathsf{A}_i} = (\cap_{i\in I} A_i)^2 \cap \beta_{\mathsf{A}_{i_0}}$.

(i) $\alpha \in \cap_{i\in I} \beta_{\mathsf{A}_i}$ implies $\alpha \in \beta_{\mathsf{A}_{i_0}}$ and $\alpha \in \beta_{\mathsf{A}_i} \subseteq A_i^2$ for all $i \in I$. Therefore, $\alpha \in \cap_{i\in I} A_i^2 = (\cap_{i\in I} A_i)^2$ or $\alpha \in \beta_{\mathsf{A}_{i_0}} \cap (\cap_{i\in I} A_i)^2$ so that $\cap_{i\in I} \beta_{\mathsf{A}_i} \subseteq (\cap_{i\in I} A_i)^2 \cap \beta_{\mathsf{A}_{i_0}}$.

(ii) $\alpha \in (\cap_{i\in I} A_i)^2 \cap \beta_{\mathsf{A}_{i_0}}$ implies $\alpha \in \beta_{\mathsf{A}_{i_0}}$. Since A_{i_0} is a sub breset of X, $\beta_{\mathsf{A}_{i_0}} = (A_{i_0})^2 \cap \beta_{\mathsf{X}}$ so that $\alpha \in \beta_{\mathsf{X}}$. Since $\cap_{i\in I} A_i^2 = (\cap_{i\in I} A_i)^2$, $\alpha \in (\cap_{i\in I} A_i)^2$ implies $\cap_{i\in I} A_i^2$ so that $\alpha \in A_i^2$ for all $i \in I$ which with the above implies $\alpha \in A_i^2 \cap \beta_{\mathsf{X}} = \beta_{\mathsf{A}_i}$ for all $i \in I$ or $\alpha \in \cap_{i\in I} \beta_{\mathsf{A}_i}$.

From (i) and (ii), we get that $\beta_{\cap_{i\in I}\mathsf{A}_i} = \cap_{i\in I} \beta_{\mathsf{A}_i} = (\cap_{i\in I} A_i)^2 \cap \beta_{\mathsf{A}_{i_0}}$.

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

By 2.3(f), it is enough to show that $\beta_{\mathsf{B}} = B^2 \cap \beta_{\cap_{i\in I}\mathsf{A}_i}$. Since B is a sub breset of A_i, $\beta_{\mathsf{B}} = B^2 \cap \beta_{\mathsf{A}_i}$ for all $i \in I$. Therefore, $B^2 \cap \beta_{\cap_{i\in I}\mathsf{A}_i} = B^2 \cap (\cap_{i\in I}\beta_{\mathsf{A}_i}) = \cap_{i\in I}(B^2 \cap \beta_{\mathsf{A}_i}) = \cap_{i\in I}\beta_{\mathsf{B}} = \beta_{\mathsf{B}}$.

Corollary 3.16 *For any breset* X, *the set* $\mathsf{S}(\mathsf{X})$ *of all sub bresets of* X *is a meet complete semi lattice.*
Proof: It follows from Lemma 3.15.

Remarks: In what follows, we show that an analogous Lemma with l-sub breset replaced by sub bresets in Lemma 3.7, is *no* longer true. In other words, for any breset X and for any family of sub bresets $(\mathsf{A}_i)_{i\in I}$ of X, the set $A = \cup_{i\in I} A_i$ together with the binary relation $\beta_{\mathsf{A}} = \cup_{i\in I}\beta_{\mathsf{A}_i}$ is a breset. However,

(1) $\mathsf{A} = \cup_{i\in I}\mathsf{A}_i$ is *not* necessarily a sub breset of X.

(2) A_i is *not* necessarily a sub breset of the breset $\mathsf{A} = \cup_{i\in I}\mathsf{A}_i$.

(3) If B is a sub breset of X such that A_i is a sub breset of B, then A is *not* necessarily a sub breset of B as can be seen in the following examples:

Example 3.17 *$A = \{a\}$, $B = \{b\}$, $A^2 = \{aa\}$, $B^2 = \{bb\}$, $X = \{a,b\}$, $\beta_{\mathsf{X}} = \{ab\}$, $\beta_{\mathsf{A}} = \phi = \beta_{\mathsf{B}}$. Then (i) A is a sub breset of X because $\beta_{\mathsf{A}} = \phi = A^2 \cap \beta_{\mathsf{X}}$ (ii) B is a sub breset of X because $\beta_{\mathsf{B}} = \phi = B^2 \cap \beta_{\mathsf{X}}$ (iii) but $\mathsf{A} \cup \mathsf{B}$ is not a sub breset of X because $\beta_{\mathsf{A}\cup\mathsf{B}} = \beta_{\mathsf{A}} \cup \beta_{\mathsf{B}} = \phi \neq \{ab\} = (A\cup B)^2 \cap \beta_{\mathsf{X}}$.*

Example 3.18 *$A = \{a,b\} = B$, $\beta_{\mathsf{A}} = \{aa\}$, $\beta_{\mathsf{B}} = \{aa,bb\}$, $A^2 = \{aa,ab,ba,bb\}$. Then A is not a sub breset of $\mathsf{A}\cup\mathsf{B}$ because $\{aa\} = \beta_{\mathsf{A}} \neq A^2 \cap (\beta_{\mathsf{A}} \cup \beta_{\mathsf{B}}) = A^2 \cap \beta_{(\mathsf{A}\cup\mathsf{B})} = \{aa,bb\}$.*

Example 3.19 *$A = \{a\}$, $B = \{b,c\}$, $C = \{a,b,c\}$, $X = \{a,b,c\}$, $\beta_{\mathsf{A}} = \{aa\}$, $\beta_{\mathsf{B}} = \{cc\}$, $\beta_{\mathsf{C}} = \{aa,ab,ba,cc\}$, $\beta_{\mathsf{X}} = \{aa,ab,ba,cc\}$. Then clearly,*

(1) $\beta_{\mathsf{A}} = A^2 \cap \beta_{\mathsf{C}} = \{aa\}$. So, A is a sub breset of C

(2) $\beta_{\mathsf{B}} = B^2 \cap \beta_{\mathsf{C}} = \{cc\}$. So, B is a sub breset of C

(3) $\beta_{\mathsf{C}} = C^2 \cap \beta_{\mathsf{X}} = \{aa,ab,ba,cc\}$. So, C is a sub breset of X

(4) $\beta_{\mathsf{A}\cup\mathsf{B}} = \beta_{\mathsf{A}} \cup \beta_{\mathsf{B}} = \{aa,cc\} \subset \{aa,ab,ba,cc\} = (A\cup B)^2 \cap \beta_{\mathsf{C}}$. So, $\mathsf{A}\cup\mathsf{B}$ is *not* a sub breset of C.

(5) A is a sub breset of C, B is a subbreset of C but $\mathsf{A}\cup\mathsf{B}$ is *not* a sub breset of C.

Lemma 3.20 *For any set indexed family of bresets $(\mathsf{A}_i)_{i\in I}$, A_{i_0} is a sub breset of the breset $\cup_{i\in I}\mathsf{A}_i$ if and only if A_{i_0} is a u-sub breset of A_i for all $i \in I$.*

Proof: A_{i_0} is a sub breset of $\cup_{i\in I}\mathsf{A}_i$ if and only if $\beta_{\mathsf{A}_{i_0}} = A_{i_0}^2 \cap (\beta_{\cup_{i\in I}\mathsf{A}_i}) = A_{i_0}^2 \cap (\cup_{i\in I}\beta_{\mathsf{A}_i}) = \cup_{i\in I}(A_{i_0}^2 \cap \beta_{\mathsf{A}_i}) = \beta_{\mathsf{A}_{i_0}} \cup (\cup_{i\neq i_0}(A_{i_0}^2 \cap \beta_{\mathsf{A}_i})) \supseteq A_{i_0}^2 \cap \beta_{\mathsf{A}_i}$ for $i \neq i_0$ if and only if A_{i_0} is a u-sub breset of A_i for all $i \in I$.

Lemma 3.21 *For any set indexed family of bresets $(\mathsf{A}_i)_{i\in I}$, A_{i_0} is a sub breset of the breset $\cap_{i\in I}\mathsf{A}_i$ if and only if A_{i_0} is an l-sub breset of A_i for all $i \in I$.*

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

Proof: A_{i_0} is a sub breset of $\cap_{i\in I}\mathsf{A}_i$ if and only if $\beta_{\mathsf{A}_{i_0}} = A_{i_0}^2 \cap (\beta_{\cap_{i\in I}\mathsf{A}_i}) = A_{i_0}^2 \cap (\cap_{i\in I}\beta_{\mathsf{A}_i}) = \cap_{i\in I}(A_{i_0}^2 \cap \beta_{\mathsf{A}_i}) = \beta_{\mathsf{A}_{i_0}} \cap (\cap_{i\neq i_0}(A_{i_0}^2 \cap \beta_{\mathsf{A}_i})) \subseteq A_{i_0}^2 \cap \beta_{\mathsf{A}_i}$ for $i \neq i_0$ if and only if A_{i_0} is an l-sub breset of A_i for all $i \in I$.

Lemma 3.22 *For any set indexed family of bresets* $(\mathsf{A}_i)_{i\in I}$, $\mathsf{A} = \cup_{i\in i}\mathsf{A}_i$ *is a breset such that*

(1) for each $i \in I$ A_i is an l-sub breset of A and (2) if B is any breset such that A_i is an l-sub breset of B for each $i \in I$ then A is an l-sub breset of B.

Proof: It follows from 3.3, 3.4 and 2.3(b).

Lemma 3.23 *For any set indexed family of bresets* $(\mathsf{A}_i)_{i\in I}$, $\mathsf{A} = \cap_{i\in i}\mathsf{A}_i$ *is a breset such that*

(1) for each $i \in I$ A is an l-sub breset of A_i and (2) if B is any breset such that B is an l-sub breset of A_i for each $i \in I$ then B is an l-sub breset of A.

Proof: It follows from 3.1, 3.2 and 2.3(b).

IV. CONCLUSION

Since bresets have an underlying set and a binary relation, notice that for homomorphisms, one can consider both functions and relations on the underlying sets, giving rise to function based homomorphisms and relation based homomorphisms. Murthy and Sujatha[15] introduces the notions of function (relational)(co,strong) homomorphism, (weak-co,full) homomorphism and studies lattice algebraic properties of images and inverse images of various sub structures of bresets under these homomorphisms in detail.

REFERENCES

1. Adamek, J., Herrlich, H., and Strecker, G.E., Abstract And Concrete Categories The Joy Of Cats.
2. Andreae, T., On the reconstruction of locally finite, infinite graphs,J. combin. Inform. System Sci. 7 (1982) 65-74 .
3. Bondy, J.A. and Hemminger, R.L.,Reconstructing infinite graphs, pacific J. Math. 52 (1974) 331-340.
4. Birkhoff, G., Lattice Theory, Amer. Math. Soc., Providence, R. I., Colloquium publications.
5. Charles Desoer and Ennest Kuh, Basic Circuit Theory, McGraw Hill, 1969.
6. Diestel, R.,Graph Theory, Electronic Edition 2000, Springer-Verlag New York 1997, 2000.
7. Finucane, H., Finite voronoi decompositions of infinite vertex transitive graphs. (arxiv.org/pdf/ 1111.0472v1.pdf).
8. Herrlich, H., and Strecker, G.E., Categori Theory : an introduction, Allyn and Bacon 1973.
9. Joe L. Mott, Abraham Kandel and Theodore P. Baker, Discrete Mathematics for Computer Scientists and Mathematicians, Prentice-Hall of India,1986.
10. Jorgen, B.J. and Gregory, G., Digraph Theory, Algorithms and Applications, Spinger-Verlag, Berlin Heidelberg New-York, 2007.
11. Kenneth H Rosen, Discrete Mathematics and Its Applications(with Combinatorics and Graph theory), Tata McGraw Hill, 2007.
12. Krishnaiyan Thulasiraman, Circuit Theory Section Editor(Five Chapters), Hand book of Electronic Engineering, Academic press, oct-2000.
13. Mac Lane. S., Categories For The Working MatheMatician, 2nd Eddition, GTM#5, Springer NY, 1998.
14. Narasingh Deo, Graph Theory With Applications to engineering and Computer Science, Prentice Hall of India, 1990.
15. Nistala V.E.S. Murthy and Lokavarapu Sujatha, Morphisms of bresets, Accepted (IJARSET).
16. Sambuc, R., Fonctions floues, Application l'aide au diagnostic en pathologie thyroidienne, Ph.D. Thesis University Marseille, France, 1975.
17. Seifter, N., On the action of nilpotent and metabelian groups on infinite, locally finite graphs, European J. Combin 10 (1989) 41-45.
18. Soardi, P.M., and woess, W., Amenability, Unimodularity, and the spectral radious of random walks on infinite graphs, Math.Z. 205 (1990) 471-486.
19. Szasz, G., An Introduction to Lattice Theory, Academic Press, New York.
20. Trembly, J.P., and Manohar, R., Discrete Mathematical Structures with Applications to Computer Science, McGraw Hill, 1975.
21. Woess, W., Amenable group actions on infinite graphs, Math. Ann. 284 (1989) 251-265.

BIOGRAPHY

Nistala V.E.S. Murthy received his M.S. in Mathematics from University of Hyderabad, Hyderabad, A.P. State, India, in 1981, Ph.D. from the University of Toledo, Toledo, OH-43606, U.S.A. in 1989 and an M.S. in Software Systems from B.I.T.S., Pilani, Rajasthan State, India, in 2000; taught a wide range of courses both in Mathematics and Computer Science in various levels and is currently professor in the department of Mathematics, AUCST, Andhra University, Vizag-53003, A.P. State, India. His areas of publications include Various Fuzzy Set Theories and Their Applications in Mathematics (Set Theory, Algebra and Topology) and Computer Science (Data

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 1, Issue 5 , December 2014

Security/Warehousing/Mining/Hiding) and Natural Language Modeling (Reprints/Preprints are Available on Request at drnvesmurthy@rediffmail.com or at http://andhrauniversity.academia.edu /NistalaVES Murthy). In his little own way, he (1) developed f-Set Theory generalizing L-fuzzy set Theory of Goguen which generalized the [0,1]-fuzzy set theory of Zadeh, the Father of Fuzzy Set Theory (2) imposed and studied algebraic/topological structures on f-sets (3) proved Representation Theorems for f-Algebraic and f-Topological objects in general.

Lokavarapu Sujatha received her M.Sc. in mathematics from Andhra University in 2008. Currently she is working on her doctoral thesis in Lattice (algebraic) properties of (inverse) images and representations of type 2 fuzzy subsets.