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An Overview of Mathematical Steady-State Modelling of Newton- Raphson Load Flow Equations Incorporating LTCT, Shunt Capacitor and FACTS Devices

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ABSTRACT*:* This paper presents an Overview of Mathematical Steady-State Modelling of Newton-RaphsonLoad Flow Equations Incorporating Load Tap-Changing Transformer, Shunt Capacitor and FACTSDevices, the incorporation of these devices expand the robustness and versatility of Newton-Raphson solution technique of load flow studies since power flow analysis of the transmission system forms the core of power system planning and operation has it provides steady state of the entire system such as real and reactive power generated and absorbed, line losses and the voltage magnitude and angles. The steady-state models of these discrete (LTCT and Shunt Capacitor) and FACTS controllers produced a set of algebraic equations which will be combined with power system network algebraic equations. The FACTS devices reviewed are Static Synchronous Series Compensator(SSSC), Thyristor Controlled Series Compensator (TCSC),Unified Power Flow Controller(UPFC), Static Synchronous Compensator (STATCOM), Interline Power Flow Controller (IPFC) and Static Var Compensator (SVC).This paper aims to provide a quick review of mathematical modelling needed for conducting load flow analysis of electrical transmission network incorporating OLTC, Shunt Capacitor and FACTS controllers.

KEYWORDS: Newton-Raphson Method, OLTC, Shunt Capacitor, SSSC, TCSC, UPFC, SVC, STATCOM, IPFC

I. INTRODUCTION

The backbone of power system planning and operation is load flow studies, it provides detail of sinusoidal steady state of the entire system –voltages, real and reactive power generated and absorbed and line losses. The steady state power and reactive powers supplied by a bus in a power network are expressed in terms of nonlinear algebraic equations and these set of nonlinear algebraic equations represent the network under steady-state conditions [1].Generally, in load flow analysis, the power system network is modeled as an electric network and solved by iterative techniques for the steady state power (real and reactive), voltages (magnitude and angles) at various buses [2].

One of such iterative technique is Newton-Raphson techniques,it approximates a set of non-linear simultaneous equations to a set of linear simultaneous equations employing Taylor's series expansion while limiting the terms to the first approximation [3]. It has convergence characteristics which are relatively powerful compared to other known iterative techniques. Also, the reliability of Newton-Raphson approach is comparatively good has it can solve cases leading to divergence with other popular processes [2]. Several devices ranging from discrete to modern power electronic devices can be incorporated into this algorithm to achieve distinct purposes such as voltage profile enhancement, system real and reactive power losses minimization and optimal economic dispatch among others.

In its basic form, power flow equation is set up thus;

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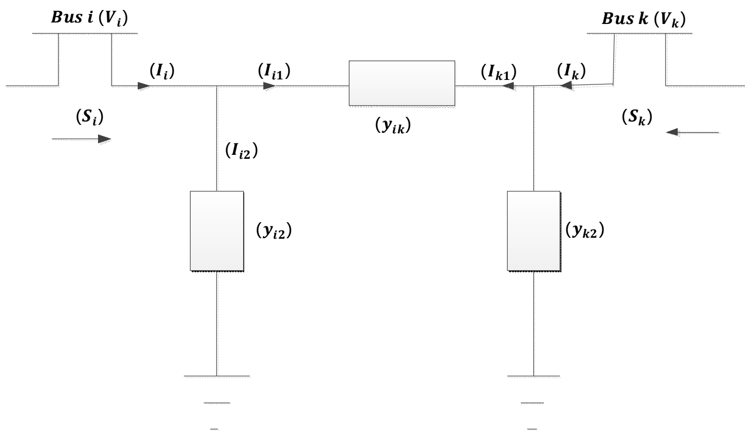

Fig.1:A line and transformer representation between two buses (Source**:** Kothari and Nagrath, 2006)

Applying KCL at bus i, the result is given inform of the equation;

$$I_i = I_{1i} + I_{2i} = (V_i - V_k)y_{ik} + V_i y_{i2} \tag{1}$$

The complex power injected by the source into bus i of the system is given by:

$$S_i = P_i + jQ_i = V_i I_i^* = P_i - jQ_i = V_i^* I_i \tag{2}$$

Substituting for I_i in equation (2) we have;

$$I_i = \frac{P_i - jQ_i}{V_i^*} = (V_i - V_k)y_{ik} + V_i y_{i2} \tag{3}$$

In general, for n number of buses we have;

$$I_i = \frac{P_i - jQ_i}{V_i^*} = V_i \sum_{j=0}^{n} y_{ij} - \sum_{j=1}^{n} y_{ij} V_j \qquad j \neq i \tag{4}$$

From equation (4), the mathematical formulation of the power flow problem results in a system of algebraic non- linear equations which must be solved by iterative techniques.Application of Newton-Raphson method to the solution of load flow equations, bus voltages and line admittances may be written in polar or rectangular form. The current I_i injected into bus i is given by equation (3) can be re-written as;

$$I_i = V_i \sum_{j=1}^{n} y_{ij} V_j \tag{5}$$

In polar form;

$$I_i = \sum_{j=1}^{n} |Y_{ij}||V_i| \angle\theta_{ij} + \delta_j \tag{6}$$

Such that the complex power at bus *i* is written thus:

$$P_i - jQ_i = V_i^* I_i = |V_i| \angle - \delta_i \sum_{j=1}^{n} |Y_{ij}||V_i| \angle\theta_{ij} + \delta_j \tag{7}$$

The Real part of the equation (7) is given as;

$$P_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \tag{7.1}$$

The imaginary part of equation (7) is given thus;

$$Q_i = -\sum_{j=1}^{n} |V_i V_j Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \tag{7.2}$$

In compact matrix form, applying Taylor's series to expand equations (7.1) and (7.2) the initial estimate gives the equation (8);

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$$
\begin{bmatrix} \Delta P_2^{(k)} \\ \vdots \\ \Delta P_n^{(k)} \\ \cdots\cdots \\ \Delta Q_2^{k} \\ \vdots \\ \Delta Q_n^{(k)} \end{bmatrix} = \left[\begin{array}{ccc:ccc} \frac{\partial P_2^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial P_2^{(k)}}{\partial \delta_n} & \frac{\partial P_2^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial P_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial P_n^{n}}{\partial \delta_n} & \frac{\partial P_n^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial P_n^{(k)}}{\partial |V_n|} \\ \hdashline \frac{\partial Q_2^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial Q_2^{(k)}}{\partial \delta_n} & \frac{\partial Q_2^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial Q_2^{(k)}}{\partial |V_n|} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n^{(k)}}{\partial \delta_2} & \cdots & \frac{\partial Q_n^{(k)}}{\partial \delta_n} & \frac{\partial Q_n^{(k)}}{\partial |V_2|} & \cdots & \frac{\partial Q_n^{(k)}}{\partial |V_n|} \end{array} \right] \begin{bmatrix} \Delta \delta_2^{(k)} \\ \vdots \\ \Delta \delta_n^{(k)} \\ \cdots\cdots \\ \Delta |V_2^{(k)}| \\ \vdots \\ \Delta |V_n^{(k)}| \end{bmatrix} \quad (8)
$$

The diagonal and off diagonal elements of the Jacobian matrix above are given by the following equations;

$$\frac{\partial P_i}{\partial |V_i|} = 2|V_i Y_{ii}| cos\theta_{ii} + \sum_{i\neq k} |V_k Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \quad (8.1)$$

$$\frac{\partial P_i}{\partial |V_i|} = -2|V_i Y_{ii}| sin\theta_{ii} - \sum_{k\neq i} |V_k Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \quad (8.2)$$

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{k=i} |V_i V_k Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \quad (8.3)$$

$$\frac{\partial P_i}{\partial \delta_k} = -|V_i V_k Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \quad (8.4)$$

$$\frac{\partial P_i}{\partial |V_k|} = |V_i Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \qquad k \neq i \quad (8.5)$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i V_k Y_{ik}| \cos(\theta_{ik} + \delta_k - \delta_i) \qquad k \neq i \quad (8.6)$$

$$\frac{\partial Q_i}{\partial |V_k|} = -|V_i Y_{ik}| \sin(\theta_{ik} + \delta_k - \delta_i) \qquad k \neq i \quad (8.7)$$

The power mismatches is given by the equation (9) and (10);

$$\Delta P_i^{(k)} = P_{i,spec} - P_{i,calc} \quad (9)$$

$$\Delta Q_i^{(k)} = Q_{i,spec} - Q_{i,calc} \quad (10)$$

The updated Voltage magnitudes and angles are given by;

$$\delta_i^{(k+1)} = \delta_i^k + \Delta\delta_i^k \quad (11)$$

$$|V_i^{(k+1)}| = |V_i^{(k)}| + \Delta |V_i^{(k)}| \quad (12)$$

A. INCORPORATION OF LOAD TAP-CHANGING TRANSFORMER (LTCT) INTO THE LOAD FLOW EQUATIONS

Load tap changingtransformers are endowed with the ability to regulate nodal voltage magnitudeautomatically by varying the transformer tap ratiounder load; they are equipped with taps on the windingto adjust either the voltage transformation or reactive flowthrough the transformer. The representation of a LTCT may be achieved bythe series connection of the short circuit admittancerepresenting a per- unit transformer and an ideal transformerwith taps ratio T :1 is reported in (4).Details of thepower flow equations incorporating Load Tap Changing Transformer with Newton Raphson iterativealgorithm is reported in (5)

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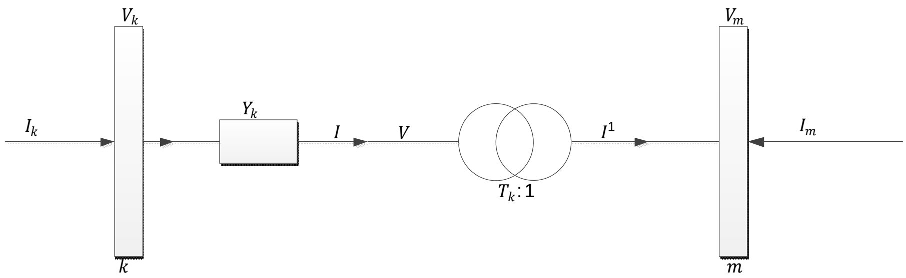

Fig.2 : Simple tap – changing transformer [4]

From Figure 1, we have,

$$\begin{bmatrix} I_k \\ I_m \end{bmatrix} = \begin{bmatrix} Y_k & -T_k Y_k \\ -T_k Y_k & T_k^2 Y_k \end{bmatrix} \begin{bmatrix} V_k \\ V_m \end{bmatrix} = \begin{bmatrix} Y_{kk} & T_k Y_{km} \\ T_k Y_{mk} & T_k^2 Y_{mm} \end{bmatrix} \begin{bmatrix} V_k \\ V_m \end{bmatrix} \tag{13}$$

The power flow equations at both ends of the transformer with T_k allowed to vary from $T_{kmin} < T_k < T_{kmax}$ are given thus;

$$P_k = V_k^2 G_{kk} + T_k V_k V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] \tag{14}$$

$$Q_k = -V_k^2 B_{kk} + T_k V_k V_m [G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)] \tag{15}$$

$$P_m = T_k^2 V_m^2 G_{mm} + T_k V_m V_k [G_{mk} \cos(\theta_m - \theta_k) + B_{mk} \sin(\theta_m - \theta_k)] \tag{16}$$

$$Q_m = -T_k^2 V_m^2 B_{mm} + T_k V_m V_k [G_{mk} \sin(\theta_m - \theta_k) - B_{mk} \cos(\theta_m - \theta_k)] \tag{17}$$

Where $Y_{kk} = Y_{mm} = G_{kk} + jB_{kk} = Y_k$, $Y_{km} = Y_{mk} = G_{km} + jB_{km} = -Y_k$, I_k is the current at bus k, I_m is the current at bus m, Y_k is the admittance at bus m, V_k is the sending end voltage magnitude at bus k, V_m is the sending end voltage magnitude at bus m, T_k is the variable tap, θ_k is the phase angle of the voltage at bus k, θ_m is the phase angle of the voltage at bus m, P_k is the active power at bus k and P_m is the active power at bus m, Q_k is the reactive power at bus k, Q_m is the reactive power at bus m, ΔP_k is the active power mismatch at bus k, ΔP_m is the active power mismatch at bus m, ΔQ_k is the reactive power mismatch at bus k and ΔQ_m is the active power mismatch at bus m.

The set of linearised power flow equations for the nodalpower injections, equations (14) – (17) assuming that theload tap changer (LTC) is controlling nodal voltagemagnitude at its Sending end (bus *k*);

$$\begin{bmatrix} \Delta P_k \\ \Delta P_m \\ \Delta Q_k \\ \Delta Q_m \end{bmatrix}^{(i)} = \begin{bmatrix} \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial \theta_m} & \frac{\partial P_k}{\partial T_m} T_k & \frac{\partial P_k}{\partial V_m} V_m \\ \frac{\partial P_m}{\partial \theta_k} & \frac{\partial P_m}{\partial \theta_m} & \frac{\partial P_m}{\partial T_m} T_k & \frac{\partial P_m}{\partial V_m} V_m \\ \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial \theta_m} & \frac{\partial Q_k}{\partial T_m} T_k & \frac{\partial Q_k}{\partial V_m} V_m \\ \frac{\partial Q_m}{\partial \theta_m} & \frac{\partial Q_m}{\partial \theta_m} & \frac{\partial Q_k}{\partial T_k} T_k & \frac{\partial Q_m}{\partial V_m} V_m \end{bmatrix} \begin{bmatrix} \Delta \theta_k \\ \Delta \theta_m \\ \frac{\Delta T_k}{T_k} \\ \frac{\Delta V_m}{V_m} \end{bmatrix} \tag{18}$$

At the end of each iteration, *i*, the tap controller is updated using the this relation defined by equation (19);

$$T_k^i = T_k^{(i-1)} + \left(\frac{\Delta T_k}{T_k}\right)^i T_1^{(i-1)} \tag{19}$$

B. MODELLING OF SHUNT CAPACITOR INTO THE LOAD FLOW EQUATIONS

Shunt capacitors are discrete controllers usually used to inject reactive power at defective buses where voltage magnitude falls outside the acceptable voltage range of 0.95p.u to 1.05 p.u [6]. This shunt capacitor could be installed near the load, in a distribution substation, along the distributionfeeder, or in a transmission substation. However, for the purpose of reactive compensation at the transmission substation both inductive and capacitive reactive types are installed [7]. The details of the power flow equations incorporating shunt capacitor with Newton Raphson iterativealgorithm is reported in [6-7].

The complex power flow equations for both active and reactive power for uncompensated transmission system solved by Newton-Raphson's iterative method are defined given below;

$$P_i - Pd_i = \sum_{j=1}^{n} |V_i V_j Y_{ij}| \cos(\theta_{ij} + \delta_j - \delta_i) \qquad i = 1,2,3 \ldots,n \tag{20}$$

$$Q_i - Qd_i = -\sum_{j=1}^{n} |V_i V_j Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \qquad i = 1,2,3 \ldots,n \tag{21}$$

Where S_i = Complex power supplied to busi^{th},I_i = Current at busi^{th}, P_i= Real power generated at bus i^{th}, Q_i= Reactive power generated at bus i^{th}, Pd_i=Real power consumed at bus i^{th}, Qd_i= Reactive power consumed at bus i^{th}, V is the bus voltage, δ is the angle associated with V, Y_{ij} is the element of bus admittance matrix, θ is the angle associated with Y_{ij} . Applying Taylor series to equation (20) and (21), the following first order approximation is obtained thus;

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \tag{23}$$

The active and reactive power mismatch is given by;

$$\Delta P_i^{(k)} = P^{sch} - P_i^{(k)} \tag{24}$$

$$\Delta Q_i^{(k)} = Q^{sch} - Q_i^{(k)} \tag{25}$$

The new estimate for bus voltages is obtained thus;

$$\delta_i^{(k+1)} = \delta_i^{(k)} + \Delta\delta_i^{(k)} \tag{26}$$

$$V_i^{(k+1)} = |V_i^k| + \Delta|V_i^k| \tag{27}$$

With injection of reactive power via shunt capacitor, the equation (20) above becomes;

$$Q_i - Qd_i + Qc_i = -\sum_{j=1}^{n} |V_i V_j Y_{ij}| \sin(\theta_{ij} + \delta_j - \delta_i) \qquad i = 1,2,3 \ldots,n \tag{28}$$

Qc_i=additional reactive power support at bus i^{th}, and it value is estimated using equation (29);

$$Q_c = P\left[\frac{1}{Pf_{(1)}} sin\left(\frac{1}{cos}(Pf_{(1)})\right) - \frac{1}{Pf_{(2)}} sin\left(\frac{1}{cos}(Pf_{(2)})\right)\right] \tag{29}$$

where P= Real Power for uncompensated system, $Pf_{(1)}$ =Uncompensated system , $Pf_{(2)}$ = Compensated system and the capacitance value required for compensation is given by;

$$C = \frac{Q_C}{2\pi f V^2} \tag{30}$$

C. INCORPORATION OF STATIC SYNCHRONOUS SERIES COMPENSATOR (SSSC) INTO THE LOAD FLOW EQUATIONS

FACTS controllers are basically power electronic devices which automatically control the parameters like line impedance, bus voltage and phase angle of the network [8]. FACTS controllers are classified as Series, Shunt, Combined series-shunt devices based on their existence in the system [9]. A member of Series controller is Static Synchronous Series Compensator (SSSC) which is used to flexibly control bus voltage magnitudes and power flows along the transmission lines. It has a voltage source converter serially connected to a transmission line through a transformer. It injects voltage in quadrature with one of the line end voltage in order to regulate the active power flow [10-11]. It does not draw reactive power from the AC system; it has its own reactive power provisions in the form of a DC capacitor [12], the equivalent circuit of SSSC is shown below. Details of thepower flow equations incorporating SSSC with Newton Raphson iterativealgorithm is reported in [11]

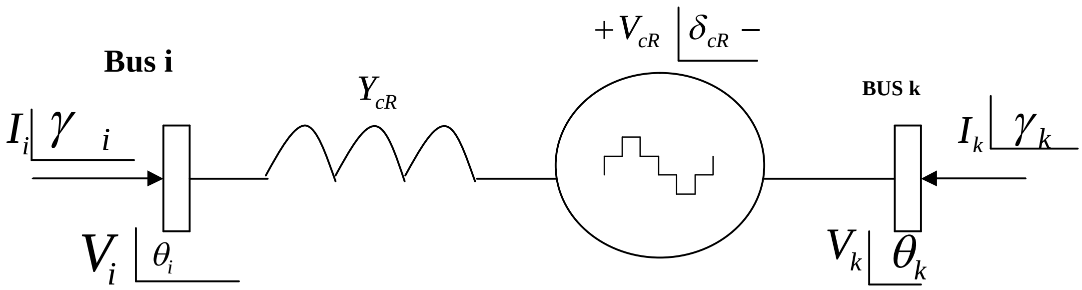

Fig. 3: SSSC equivalent circuit [10, 11]

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From [12], the SSSC voltage source is given by the equation;

$$E_{CR}^{\rho} = V_{CR}^{\rho}(cos\delta_{CR}^{\rho} + jsin\delta_{CR}^{\rho} \tag{31}$$

The boundary condition for V_{CR} and δ_{CR} are as given in equations (32) and (33);

$$V_{CR\,(\min)} \leq V_{CR} \leq V_{CR(\max)} \tag{32}$$

$$0 \leq \delta_{CR} \leq 2\pi \tag{33}$$

With the SSSC Thevenin equivalent circuit of fig. 2 and equations (31) to (33), the expressions for the active and reactive powers at bus k are written thus;

$$P_i = V_i^2 G_{ii} - V_i V_k[G_{ik}cos(\theta_i - \theta_k) - B_{ik}sin(\theta_i - \theta_k)] - V_i V_{CR}[G_{ik}cos(\theta_i - \delta_{CR}) - B_{ik}\sin(\theta_i - \delta_{CR})] \tag{34}$$

$$Q_i = V_i^2 B_{ii} - V_i V_k[G_{ik}sin(\theta_i - \theta_k) - B_{ik}cos(\theta_i - \theta_k)] - V_i V_{CR}[G_{ik}sin(\theta_i - \delta_{CR}) - B_{ik}\cos(\theta_i - \delta_{CR})] \tag{35}$$

For the converter, the active and reactive powers are given by the equations;

$$P_{CR} = V_{CR}^2 G_{ii} - V_{CR} V_i[G_{ik}cos(\delta_{CR} - \theta_i) - B_{ik}sin(\delta_{CR} - \theta_i)] - V_{CR} V_i[G_{ik}cos(\delta_{CR} - \theta_k) - B_{ik}\sin(\delta_{CR} - \theta_i)] \tag{36}$$

$$Q_{CR} = -V_{CR}^2 B_{kk} - V_{CR} V_k[G_{ik}sin(\delta_{CR} - \theta_k) - B_{ik}cos(\delta_{CR} - \theta_k)] - V_{CR} V_k[G_{ik}sin(\delta_{CR} - \theta_k) - B_{ik}\cos(\delta_{CR} - \theta_k)] \tag{37}$$

With the incorporation of SSSC, system admittance matrix and conventional Jacobian matrix is formed thus;

$$\begin{bmatrix} \Delta P_i \\ \Delta P_k \\ \Delta Q_i \\ \Delta Q_k \\ \Delta P_{ik} \\ \Delta Q_{ik} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial \theta_k} & \frac{\partial P_i}{\partial V_i}V_i & \frac{\partial P_i}{\partial V_k}V_k & \frac{\partial P_i}{\partial \delta_{CR}} & \frac{\partial P_i}{\partial V_{CR}}V_{CR} \\ \frac{\partial P_k}{\partial \theta_i} & \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial V_i}V_i & \frac{\partial P_k}{\partial V_k} & \frac{\partial P_k}{\partial \delta_{CR}} & \frac{\partial P_k}{\partial V_{CR}}V_{CR} \\ \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial \theta_k} & \frac{\partial Q_i}{\partial V_i}V_i & \frac{\partial Q_i}{\partial V_k} & \frac{\partial Q_i}{\partial \delta_{CR}} & \frac{\partial Q_i}{\partial V_{CR}}V_{CR} \\ \frac{\partial Q_k}{\partial \theta_i} & \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial V_i}V_i & \frac{\partial Q_k}{\partial V_k} & \frac{\partial Q_k}{\partial \delta_{CR}} & \frac{\partial Q_k}{\partial V_{CR}}V_{CR} \\ \frac{\partial P_{ik}}{\partial \theta_i} & \frac{\partial P_{ik}}{\partial \theta_k} & \frac{\partial P_{ik}}{\partial V_i}V_i & \frac{\partial P_{ik}}{\partial V_k} & \frac{\partial P_{ik}}{\partial \delta_{CR}} & \frac{\partial P_{ik}}{\partial V_{CR}}V_{CR} \\ \frac{\partial Q_{ik}}{\partial \theta_i} & \frac{\partial Q_{ik}}{\partial \theta_k} & \frac{\partial Q_{ik}}{\partial V_i}V_i & \frac{\partial Q_{ik}}{\partial V_k} & \frac{\partial Q_{ik}}{\partial \delta_{CR}} & \frac{\partial Q_{ik}}{\partial V_{CR}}V_{CR} \end{bmatrix} \begin{bmatrix} \Delta\theta_i \\ \Delta\theta_k \\ \frac{\Delta V_i}{V_i} \\ \frac{\Delta V_k}{\Delta V_k} \\ \Delta\delta_{CR} \\ \frac{\Delta V_{CR}}{V_{CR}} \end{bmatrix} \tag{38}$$

D. INCORPORATION OF THYRISTOR CONTROLLED SERIES COMPENSATOR (TCSC) INTO THE LOAD FLOW EQUATIONS

Thyristor Controlled Series Compensator (TCSC) are FACTS controllers based on thyristor controlled reactor (TCRs), it permits rapid and continuous changes of transmission impedance, controlling power flow in the line and improving system stability[12].TCSC belongs to the family of FACTS controllers that are used for enhancing dynamic performance of power systems in terms of voltage/angle stability while improving the power transfer capability and voltage profile in steady-state conditions [13-15]. The TCSC moduleis shown below in fig.3 and the details of thepower flow equations incorporating TCSC with Newton Raphson iterativealgorithm is reported in [12, 18].

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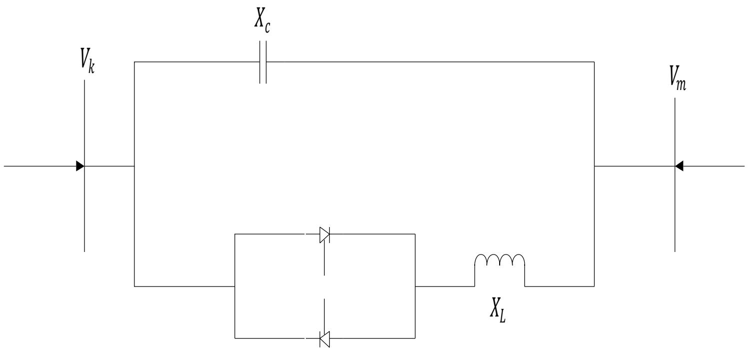

Fig.4: TCSC module connected between two buses [12, 16]

The fundamental TCSC equivalent reactance is expressed by the equation (29) below;

$$X_{TCSC} = -X_C + \frac{X_C+X_{LC}}{\pi}\{2(\pi-\alpha)+\sin[2(\pi-\alpha)]\} - \frac{4X_{LC}^2}{X_L(\pi)}cos^2(\pi-\alpha)\left\{\left(\frac{X_C}{X_L}\right)^{\frac{1}{2}}tan\left[\left(\frac{X_C}{X_L}\right)^{\frac{1}{2}}((\pi-\alpha)\right] - tan(\pi-\alpha)\right\} \quad (39)$$

Where $X_{Lc} = \frac{X_C X_L}{X_C - X_L}$

The active power equations at bus k and m are given equations (40) and (41) below;

$$P_K = V_k V_m B_{km}\sin(\theta_k-\theta_m) \quad (40)$$

$$P_m = V_m V_k B_{mk}\sin(\theta_m-\theta_k) \quad (41)$$

The reactive power equations at bus k and m are given equations (42) and (43) below;

$$Q_K = -V_k^2 B_{kk} - V_k V_m B_{km}\cos(\theta_k-\theta_m) \quad (42)$$

$$Q_m = -V_m^2 B_{mm} - V_m V_k B_{mk}\cos(\theta_m-\theta_k) \quad (43)$$

Where $B_{kk} = -B_{km},\ \ B_{mm} = -B_{mk}\ \ and\ B_{TCSC} = \frac{1}{X_{TCSC}}$

With TCSC controlling active power flowing from bus k to bus m at a specified value, the set of linearised power flow equations is given thus;

$$\begin{bmatrix}\Delta P_k\\ \Delta P_m\\ \Delta Q_k\\ \Delta Q_m\\ \Delta P_{km}^{\alpha TCSC}\end{bmatrix} = \begin{bmatrix} \frac{\partial P_k}{\partial\theta_k} & \frac{\partial P_k}{\partial\theta_m} & \frac{\partial P_k}{\partial V_k}V_k & \frac{\partial P_k}{\partial V_m}V_m & \frac{\partial P_K}{\partial\alpha}\\ \frac{\partial p_m}{\partial\theta_k} & \frac{\partial P_m}{\partial\theta_m} & \frac{\partial P_m}{\partial V_k}V_k & \frac{\partial P_m}{\partial V_m}V_m & \frac{\partial P_m}{\partial\alpha}\\ \frac{\partial Q_k}{\partial\theta_k} & \frac{\partial Q_k}{\partial\theta_m} & \frac{\partial Q_k}{\partial V_k}V_k & \frac{\partial Q_k}{\partial V_m}V_m & \frac{\partial Q_k}{\partial\alpha}\\ \frac{\partial Q_m}{\partial\theta_k} & \frac{\partial Q_m}{\partial\theta_m} & \frac{\partial Q_m}{\partial V_k}V_k & \frac{\partial Q_m}{\partial V_m}V_m & \frac{\partial Q_m}{\partial\alpha}\\ \frac{\partial P_{km}^{\alpha TCSC}}{\partial\theta_k} & \frac{\partial P_{km}^{\alpha TCSC}}{\partial\theta_m} & \frac{\partial P_{km}^{\alpha TCSC}}{\partial V_k}V_k & \frac{\partial P_{km}^{\alpha TCSC}}{\partial V_m} & \frac{\partial P_{km}^{\alpha TCSC}}{\partial\alpha} \end{bmatrix} \begin{bmatrix}\Delta\theta_k\\ \Delta\theta_m\\ \frac{\Delta V_k}{V_k}\\ \frac{\Delta V_m}{V_m}\\ \Delta\alpha\end{bmatrix} \quad (44)$$

Where $\Delta P, \Delta Q, \Delta P_{km}^{\propto TCSC}$ are power mismatch equation expressed as;

$$\Delta P_k = P_{GK} - P_{LK} - P_K^{Cal} = P_k^{Sch} - P_k^{Cal} = 0 \quad (45)$$

$$\Delta Q_k = Q_{GK} - Q_{LK} - Q_K^{Cal} = Q_k^{Sch} - Q_k^{Cal} = 0 \quad (46)$$

$$\Delta P_{km}^{\alpha TCSC} = P_{km}^{reg} - P_{km}^{\alpha TCSC,Cal} \quad (47)$$

P_{km}^{reg} = The active power to be controlled from bus k to bus m,

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Similarly $\Delta\theta, \Delta V, \Delta\alpha^{TCSC}$ are the state variable expressed as;

$$\Delta\theta = \theta^{i+1} - \theta^{i} \quad (48)$$

$$\Delta V = V^{i+1} - V^{i} \quad (49)$$

$$\Delta\alpha^{TCSC} = \alpha^{TCSC(i+1)} - \alpha^{TCSC(i)} \quad (50)$$

$\Delta\alpha^{TCSC}$is the incremental change in the TCSC firing angle at the i^th iteration,The Jacobian elements for the series reactance, as a function of the firing angle α *TCSC* are given below. Partial derivatives of the variable series impedance model are:

$$\frac{\delta P_k}{\delta X} X = -V_k V_m B_{km} \sin(\theta_k - \theta_m) \quad (51)$$

$$\frac{\delta Q_k}{\delta X} X = V_k^2 B_{kk} + V_k V_m B_{km} \cos(\theta_k - \theta_m) \quad (52)$$

$$\frac{\delta P_{km}}{\delta X} = \frac{\delta P_k}{\delta X} X \quad (53)$$

Partial derivatives of the firing angle model are given by:

$$\frac{\delta P_k}{\delta\alpha} = P_k B_{TCSC} \frac{\delta X_{TCSC}}{\delta\alpha} \quad (54)$$

$$\frac{\delta Q_k}{\delta\alpha} = Q_k B_{TCSC} \frac{\delta X_{TCSC}}{\delta\alpha} \quad (55)$$

$$\frac{\delta B_{TCSC}}{\delta\alpha} = B_{TCSC}^2 \frac{\delta X_{TCSC}}{\delta\alpha} \quad (56)$$

$$\frac{\delta X_{TCSC}}{\delta\alpha} = -\left(\frac{X_C + X_{LC}}{\pi}\right)\left[\{2(\pi-\alpha) + \sin[2(\pi-\alpha)]\} - \frac{4X_{LC}^2}{X_L(\pi)} cos^2(\pi-\alpha)\left\{\left(\frac{X_C}{X_L}\right)^{\frac{1}{2}} tan\left[\left(\frac{X_C}{X_L}\right)^{\frac{1}{2}}((\pi-\alpha)\right] - tan(\pi-\alpha)\right\}\right] \quad (57)$$

E. INCORPORATION OF UNIFIED POWER FLOW CONTROLLER(UPFC) INTO THE LOAD FLOW EQUATIONS

Unified power flow controller is ashunt-series compensator; it operates as a combination of voltage regulator, variable series compensator and phase shifter [16]. UPFChas the ability to control, simultaneously orselectively, all the parameters affecting power flow in thetransmission line (voltage, impedance, and phase angle) and as a result, it can fulfill functions of reactive shunt compensation, series compensation and phase shifting meeting multiple control objectives [16-18]. The shunt converter of the UPFC controls the UPFC busvoltage/shunt reactive power and the dc link capacitor voltage while the series converter of the UPFC controls the transmission linereal/active power flows by injecting a series voltage of adjustablemagnitude and phase angle [18-21].

The configuration of UPFCis shown below in fig. 4 and the details of thepower flow equations incorporating UPFC with Newton Raphson iterativealgorithm is reported in [16, 18].

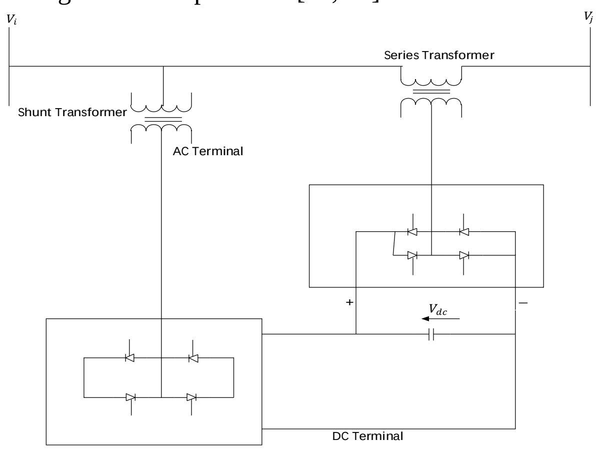

Fig. 5: Configuration of UPFC [18]

The UPFC voltage sources are given by the equations [58, 59] below;

$$E_{VR} = V_{VR}(cos\delta_{VR} + jsin\delta_{VR} \quad (58)$$

$$E_{VR} = V_{CR}(cos\delta_{CR} + jsin\delta_{CR} \quad (59)$$

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WhereV_{VR} andδ_{VR} are the controllable magnitude ($V_{VR\,min} \leq V_{VR} \leq V_{VR,max}$) and phase angle ($0 \leq \delta_{VR} \leq 2\pi$) of the voltage source representing the shunt converter and also the magnitude V_{CR} andphase angle δ_{CR} of the voltage source representing the series converter are controlled between limits ($V_{CR\,min} \leq V_{CR} \leq V_{CR,max}$) and ($0 \leq \delta_{VR} \leq 2\pi$) respectively.The active and reactive power equations are at bus k are given by equations (60) and (61) respectively;

$$P_k = V_k^2 G_{kk} + V_k V_m[G_{km} cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)] + V_k V_{CR}[G_{km} cos(\theta_k - \delta_{CR}) + B_{km} \sin(\theta_k - \delta_{CR})] + V_k V_{VR}[G_{VR} cos(\theta_k - \delta_{VR}) + B_{VR} \sin(\theta_k - \delta_{VR})] \quad (60)$$

$$Q_k = -V_k^2 B_{kk} + V_k V_m[G_{km} sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)] + V_k V_{CR}[G_{km} sin(\theta_k - \delta_{CR}) - B_{km} \cos(\theta_k - \delta_{CR})] + V_k V_{VR}[G_{VR} sin(\theta_k - \delta_{VR}) - B_{VR} \cos(\theta_k - \delta_{VR})] \quad (61)$$

At bus m, the active and reactive power equations are given by the equations below;

$$P_k = V_m^2 G_{mm} + V_m V_k[G_{mk} cos(\theta_m - \theta_k) + B_{mk} \sin(\theta_m - \theta_k)] + V_m V_{CR}[G_{mm} cos(\theta_m - \delta_{CR}) + B_{mm} \sin(\theta_m - \delta_{CR})] \quad (62)$$

$$Q_k = V_m^2 B_{mm} + V_m V_k[G_{mk} sin(\theta_m - \theta_k) - B_{mk} \cos(\theta_m - \theta_k)] + V_m V_{CR}[G_{mm} sin(\theta_m - \delta_{CR}) + B_{mm} \cos(\theta_m - \delta_{CR})] \quad (63)$$

The active and reactive power equations for the Series converter:

$$P_{CR} = V_{CR}^2 G_{mm} + V_{CR} V_k[G_{km} cos(\delta_{CR} - \theta_k) + B_{km} sin(\delta_{CR} - \theta_k)] + V_{CR} V_m[G_{mm} \cos((\delta_{CR} - \theta_m) + B_{mm} sin(\delta_{CR} - \theta_m)] \quad (64)$$

$$Q_{CR} = -V_{CR}^2 B_{mm} + V_{CR} V_k[G_{km} sin(\delta_{CR} - \theta_k) - B_{km} cos(\delta_{CR} - \theta_k)] + V_{CR} V_m[G_{mm} \sin((\delta_{CR} - \theta_m) - B_{mm} sin(\delta_{CR} - \theta_m)] \quad (65)$$

The active and reactive power equations for the Shunt converter:

$$P_{VR} = -V_{VR}^2 G_{VR} + V_{VR} V_k[G_{VR} \cos(\delta_{VR} - \theta_k) + B_{VR} \sin(\delta_{VR} - \theta_k)] \quad (66)$$

$$Q_{VR} = -V_{VR}^2 B_{VR} + V_{VR} V_k[G_{VR} \sin(\delta_{VR} - \theta_k) + B_{VR} \cos(\delta_{VR} - \theta_k)] \quad (67)$$

If UPFC is to control these following parameters: Voltage magnitude at the shunt converter terminal (bus k), active power flow from bus m to bus k, reactive power injected at bus m andtaking bus m to be a PQ bus, then the linearized system of equation is as follow;

$$\begin{bmatrix} \Delta P_k \\ \Delta P_m \\ \Delta Q_k \\ \Delta Q_m \\ \Delta P_{mk} \\ \Delta Q_{mk} \\ \Delta P_{bb} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_{k(}}{\partial \theta_k} & \frac{\partial P_k}{\partial \theta_m} & \frac{\partial P_k}{\partial V_{VR}} V_{VR} & \frac{\partial P_k}{\partial V_m} V_m & \frac{\partial P_k}{\partial \delta_{CR}} & \frac{\partial P_k}{\partial V_{CR}} V_{CR} & \frac{\partial P_k}{\partial \delta_{VR}} \\ \frac{\partial P_m}{\partial \theta_k} & \frac{\partial P_m}{\partial \theta_m} & 0 & \frac{\partial P_m}{\partial V_m} V_m & \frac{\partial P_m}{\partial \delta_{CR}} & \frac{\partial P_m}{\partial V_{CR}} V_{CR} & 0 \\ \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial \theta_m} & \frac{\partial Q_k}{\partial V_{CR}} V_{VR} & \frac{\partial Q_k}{\partial V_{VR}} V_m & \frac{\partial Q_k}{\partial \delta_{CR}} & \frac{\partial Q_m}{\partial V_{CR}} V_{CR} & \frac{\partial Q_k}{\partial \delta_{VR}} \\ \frac{\partial Q_m}{\partial \theta_k} & \frac{\partial Q_m}{\partial \theta_m} & 0 & \frac{\partial Q_m}{\partial V_m} V_m & \frac{\partial Q_m}{\partial \delta_{CR}} & \frac{\partial Q_m}{\partial V_{CR}} V_{CR} & 0 \\ \frac{\partial P_{mk}}{\partial \theta_k} & \frac{\partial P_{mk}}{\partial \theta_m} & 0 & \frac{\partial P_{mk}}{\partial V_m} V_m & \frac{\partial P_{mk}}{\partial \delta_{CR}} & \frac{\partial P_{mk}}{\partial V_{CR}} V_{CR} & 0 \\ \frac{\partial Q_{mk}}{\partial \theta_k} & \frac{\partial Q_{mk}}{\partial Q_m} & 0 & \frac{\partial Q_{mk}}{\partial V_m} V_m & \frac{\partial Q_{mk}}{\partial \delta_{CR}} & \frac{\partial Q_{mk}}{\partial \delta_{CR}} V_{CR} & 0 \\ \frac{\partial P_{bb}}{\partial \theta_k} & \frac{\partial P_{bb}}{\partial \theta_m} & \frac{\partial P_{bb}}{\partial V_{VR}} V_{VR} & \frac{\partial P_{bb}}{\partial V_m} V_m & \frac{\partial P_{bb}}{\partial \delta_{CR}} & \frac{\partial P_{bb}}{\partial V_{CR}} V_{CR} & \frac{\partial P_{bb}}{\partial \delta_{VR}} \end{bmatrix} \begin{bmatrix} \Delta \theta_k \\ \Delta \theta_m \\ \frac{\Delta V_{VR}}{V_{VR}} \\ \frac{\Delta V_m}{V_m} \\ \Delta \delta_{CR} \\ \frac{\Delta V_{CR}}{V_{CR}} \\ \Delta \delta_{VR} \end{bmatrix} \quad (68)$$

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F. INCORPORATION OF STATIC SYNCHRONOUS COMPENSATOR(STATCOM) INTO THE LOAD FLOW EQUATIONS

Static Synchronous Compensator (STATCOM) is shunt type reactive power compensating FACTS controller capable of generating or absorbing reactive power and most widely used of all the Voltage Sourced Controller (VSC) [10, 22]. It consists of voltage source inverters connected to an energy storage device on one side and to the power system on the other side. Also, at the output, STATCOM produces less harmonic content and higher compensation Volt Ampere (VA) capacity. Inclusion of STATCOM on power system can appreciably improvedynamic voltage control in transmission and distribution,power oscillation damping in transmission system,transient stability,voltage flicker control and active power control in the connected lines[10].
The Thevenin equivalent circuit representing the fundamental frequency operation of theswitched-mode voltage sourced converter and its transformer and as well as the the details of thepower flow equations incorporating STATCOM with Newton Raphson iterativealgorithm is reported in [10, 22-23]

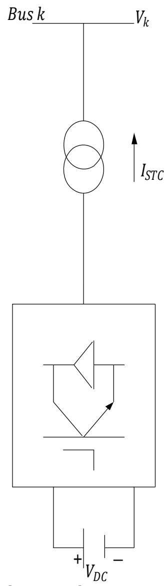

Fig.6a: STATCOM schematic diagram

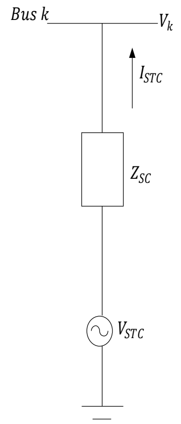

Fig.6b: STATCOM equivalent circuit

Figure 6: Thevenin Equivalent Circuit Diagram of STATCOM [10, 22-23]

$V_{STC} = V_k + Z_{SC} I_{STC}$ (69)

Expressed in Norton equivalent form;

$I_{STC} = I_N - Y_{SC} V_k$ (70)

Where $I_N = Y_{SC} V_{STC}$ such that the bound constraint on the STATCOM voltage injection V_{STC}

$V_{STC\,min} \leq V_{STC} \leq V_{STC\,max}$ (71)

In the above equations, $V_k$represents bus k voltage and V_{STC} represents the voltage source inverter.$I_N$is the Norton's current while $I_{STC}$is the inverter's current. Also,$Z_{SC}$and $Y_{SC}$are the transformer's impedance and short-circuit admittance respectively. The current expression in (60) is transformed into a power expression by the VSC and power injected into bus *k* thus;

$S_{STC} = V_{STC} I_{STC}^* = V_{STC}^2 Y_{SC}^* - V_{STC} Y_{SC}^* V_k^*$ (72)

$S_k = V_k I_{STC}^* = V_{STC} Y_{SC}^* V_k^* - V_k^2 Y_{SC}^*$ (73)

Using rectangular coordinate representation,

$V_k = e_k + j f_k$ (74)

$V_{STC} = e_{STC} + j f_{STC}$ (75)

$|V_{STC}| = \sqrt{e_{STC}^2 + f_{STC}^2}$ (76)

$\delta_{STC} = \frac{1}{tan\left(\frac{f_{STC}}{e_{STC}}\right)}$ (77)

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Where $|V_{STC}|$= voltage magnitude, δ_{STC}= voltage angle, e_k *and* f_k are the real and imaginary parts of the bus voltage, also e_{STC} and f_{STC} are the real and imaginary parts of the STATCOM voltage respectively. The active and reactive powers for the STATCOM and node *k* respectively are given below;

$$P_{STC} = G_{SC}[(e_{STC}^2 + f_{STC}^2) - (e_{STC}e_k + f_{STC}f_k)] + B_{SC}(e_{STC}f_k - f_{STC}e_k) \quad (78)$$

$$Q_{STC} = G_{SC}(e_{STC}e_k - f_{STC}e_k) + B_{SC}(-e_{STC}^2 - f_{STC}^2 + e_{STC}e_k + f_{STC}f_k) \quad (79)$$

And also,

$$P_k = G_{SC}[(e_k^2 + f_k^2) - (e_k e_{STC} + f_k f_{STC})] + B_{SC}(e_k f_{STC} - f_k e_{STC}) \quad (80)$$

$$Q_k = G_{SC}(e_k f_{STC} - f_k e_{STC}) + B_{SC}[(e_k f_{STC} + f_k e_{STC}) - (e_k^2 + f_k^2)] \quad (81)$$

The set of linearised power flow equation for the complete system is given by;

$$\begin{bmatrix} \Delta P_k \\ \Delta|V_k|^2 \\ \Delta P_{STC} \\ \Delta Q_{STC} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_k}{\partial e_k} & \frac{\partial P_k}{\partial f_k} & \frac{\partial P_k}{\partial e_{STC}} & \frac{\partial P_k}{\partial f_{STC}} \\ \frac{\partial |V_k|^2}{\partial e_k} & \frac{\partial |V_k|^2}{\partial f_k} & 0 & 0 \\ \frac{\partial P_{STC}}{\partial e_k} & \frac{\partial P_{STC}}{\partial f_k} & \frac{\partial P_{STC}}{\partial e_{STC}} & \frac{\partial P_{STC}}{\partial f_{STC}} \\ \frac{\partial Q_{STC}}{\partial e_k} & \frac{\partial Q_{STC}}{\partial f_k} & \frac{\partial Q_{STC}}{\partial e_{STC}} & \frac{\partial Q_{STC}}{\partial f_{STC}} \end{bmatrix} \begin{bmatrix} \Delta e_k \\ \Delta f_k \\ \Delta e_{STC} \\ \Delta f_{STC} \end{bmatrix} \quad (82)$$

The partial derivatives for the STATCOM model are given below;

$$\frac{\delta P_k}{\delta e_k} = G_{SC}(2e_k - e_{STC}) + B_{SC}f_{STC} \quad (82.1)$$

$$\frac{\delta P_k}{\delta f_k} = G_{SC}(2e_f - f_{STC}) - B_{SC}e_{STC} \quad (82.2)$$

$$\frac{\delta V_k}{\delta e_k} = \frac{e_k}{\sqrt{e_k^2+f_k^2}} \quad (82.3)$$

$$\frac{\delta V_k}{\delta f_k} = \frac{f_k}{\sqrt{e_k^2+f_k^2}} \quad (82.4)$$

$$\frac{\delta P_k}{\delta e_{STC}} = -G_{SC}e_k - B_{SC}f_k \quad (82.5)$$

$$\frac{\delta P_i}{\delta f_{STC}} = -G_{SC}f_i + B_{SC}e_i \quad (82.6)$$

$$\frac{\delta P_{STC}}{\delta e_i} = -G_{SC}e_{STC} - B_{SC}f_{STC} \quad (82.7)$$

$$\frac{\delta P_{STC}}{\delta F_i} = -G_{SC}F_{STC} + B_{SC}E_{STC} \quad (82.8)$$

$$\frac{\delta P_{STC}}{\delta e_{STC}} = G_{SC}(2e_{STC} - e_i) + B_{SC}f_i \quad (82.9)$$

$$\frac{\delta P_{STC}}{\delta F_{STC}} = G_{SC}(2F_{STC} - F_i) - B_{SC}E_i \quad (82.10)$$

$$\frac{\delta Q_{STC}}{\delta e_i} = -G_{SC}f_{STC} + B_{SC}e_{STC} \quad (82.11)$$

$$\frac{\delta Q_{STC}}{\delta f_i} = G_{SC}e_{STC} + B_{SC}f_{STC} \quad (82.12)$$

$$\frac{\delta Q_{STC}}{\delta e_{STC}} = G_{SC}f_i - B_{SC}(2e_{STC} - e_i) \quad (82.13)$$

$$\frac{\delta Q_{STC}}{\delta f_{STC}} = -G_{SC}e_i - B_{SC}(2f_{STC} - f_i) \quad (82.14)$$

G. INCORPORATION OFINTER LINE POWER FLOW CONTROLLER (IPFC)INTO THE LOAD FLOW EQUATIONS

IPFC is one latest generation of FACTS controller it comprises a number of Static Synchronous Series Compensators (SSSC) and it significantly extends the control of power flows of multi lines and as well as the concept of voltage beyond what is achievable by the one-converter FACTS controller. The simplest IPFC consist of two back-to-back dc-to-ac converters, which are connected in series with two transmission lines through series coupling transformers and the dc terminals of the converters are connected together via a common dc link [24].The schematic representation of IPFC with two converters represented by two controllable series-injected voltage sources and as well as thedetails of the power flow equations incorporating IPFC with Newton Raphson iterative algorithm is reported in [24]

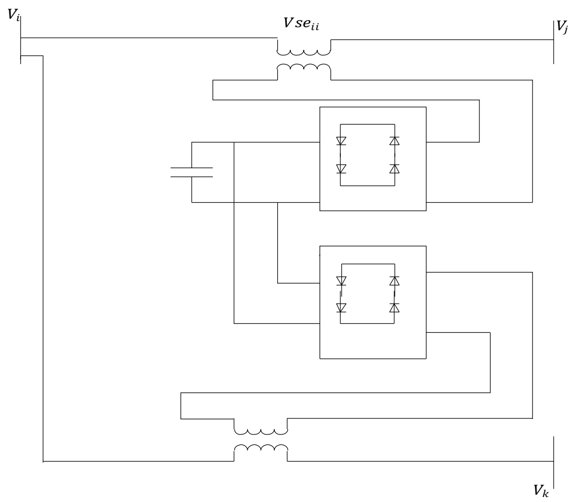

Figure 7:Operational principle of two converters IPFC [24]

The power flowequations for IPFC can be derived thus;

$$P_i = V_i^2 G_{ii} - \sum_n V_i V_n [G_{in}\cos(\theta_i - \theta_n) + B_{in} sin(\theta_i - \theta_n)] - \sum_n V_i Vse_{in}[G_{in} cos(\theta_i - \theta_{se\ in}) + B_{in} sin(\theta_i - \theta_{se\ in})] \quad (83)$$

$$Q_i = -V_i^2 B_{ii} - \sum_n V_i V_n [G_{in}\sin(\theta_i - \theta_n) - B_{in} cos(\theta_i - \theta_n)] - \sum_n V_i Vse_{in}[G_{in} sin(\theta_i - \theta_{se\ in}) - B_{in} cos(\theta_i - \theta_{se\ in})] \quad (84)$$

$$P_{ni} = V_n^2 G_{nn} - V_i V_n [G_{in}\cos(\theta_n - \theta_i) + B_{in} sin(\theta_n - \theta_i)] + V_j Vse_{in}\left[G_{in} cos\left(\theta_j - \theta_{se\ in}\right) + B_{in} sin\left(\theta_j - \theta_{se\ in}\right)\right] \quad (85)$$

$$Q_i = -V_n^2 B_{nn} - V_i V_j [G_{in}\sin(\theta_n - \theta_i) - B_{in} cos(\theta_n - \theta_i)] + V_n Vse_{in}[G_{in} sin(\theta_n - \theta_{se\ in}) - B_{in} cos(\theta_n - \theta_{se\ in})] \quad (86)$$

Where $G_{in} + jB_{in} = \frac{1}{Z_{se\ in}}$, $G_{ii} = \sum_n G_{in}$, $B_{ii} = \sum_n B_{in}$, $G_{nn} + jB_{nn} = \frac{1}{Z_{se\ in}}$,

The constraint on the equivalent controllable injected voltage source magnitude and angle of the series converter are given by;

$$Vse_{in}^{min} \le Vse_{in} \le Vse_{in}^{max} \quad (87)$$

$$-\Pi \le \theta se_{in} \le \Pi \quad (88)$$

Where Vse_{in}^{min} and Vse_{in}^{max} are the minimum and maximum voltage limits ofVse_{in} respectively, the active and reactive

$$P_{ni} - P_{ni}^{spec} = 0 \quad (89)$$

$$Q_{ni} - Q_{ni}^{spec} = 0 \quad (90)$$

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$$
\begin{bmatrix} P_{ji}^{spec}-P_{ji} \\ Q_{ji}^{spec}-Q_{ji} \\ P_{ki}-P_{ki} \\ PE \\ \Delta P_i \\ \Delta Q_i \\ \Delta P_j \\ \Delta Q_j \\ \Delta P_k \\ \Delta Q_k \end{bmatrix}
=
\begin{bmatrix}
\frac{\partial P_{ji}}{\partial \theta se_{ji}} & \frac{\partial P_{ji}}{\partial Vse_{ji}} & 0 & 0 & \frac{\partial P_{ji}}{\partial \theta_i} & \frac{\partial P_{ji}}{\partial V_i} & \frac{\partial P_{ji}}{\partial \theta_j} & \frac{\partial P_{ji}}{\partial V_j} & 0 & 0 \\
\frac{\partial Q_{ji}}{\partial \theta se_{ji}} & \frac{\partial Q_{ji}}{\partial Vse_{ji}} & 0 & 0 & \frac{\partial Q_{ji}}{\partial \theta_i} & \frac{\partial Q_{ji}}{\partial V_i} & \frac{\partial Q_{ji}}{\partial \theta_j} & \frac{\partial Q_{ji}}{\partial V_j} & 0 & 0 \\
0 & 0 & \frac{\partial P_{ki}}{\partial \theta se_{ki}} & \frac{\partial P_{ki}}{\partial Vse_{ki}} & \frac{\partial P_{ki}}{\partial \theta_i} & \frac{\partial P_{ki}}{\partial V_i} & 0 & 0 & \frac{\partial P_{ki}}{\partial \theta_k} & \frac{\partial P_{ki}}{\partial V_k} \\
\frac{\partial PE}{\partial \theta se_{ji}} & \frac{\partial PE}{\partial Vse_{ji}} & \frac{\partial PE}{\partial \theta se_{ki}} & \frac{\partial PE}{\partial Vse_{ki}} & \frac{\partial PE}{\partial \theta_i} & \frac{\partial PE}{\partial V_i} & \frac{\partial PE}{\partial \theta_j} & \frac{\partial PE}{\partial V_j} & \frac{\partial PE}{\partial \theta_k} & \frac{\partial PE}{\partial V_k} \\
\frac{\partial P_i}{\partial \theta se_{ji}} & \frac{\partial P_i}{\partial Vse_{ji}} & \frac{\partial P_i}{\partial \theta se_{ki}} & \frac{\partial P_i}{\partial Vse_{ki}} & \frac{\partial P_i}{\partial \theta_i} & \frac{\partial P_i}{\partial V_i} & \frac{\partial P_i}{\partial \theta_j} & \frac{\partial P_i}{\partial V_j} & \frac{\partial P_i}{\partial \theta_k} & \frac{\partial P_i}{\partial V_k} \\
\frac{\partial Q_i}{\partial \theta se_{ji}} & \frac{\partial Q_i}{\partial Vse_{ji}} & \frac{\partial Q_i}{\partial \theta se_{ki}} & \frac{\partial Q_i}{\partial Vse_{ki}} & \frac{\partial Q_i}{\partial \theta_i} & \frac{\partial Q_i}{\partial V_i} & \frac{\partial Q_i}{\partial \theta_j} & \frac{\partial Q_i}{\partial V_j} & \frac{\partial Q_i}{\partial \theta_k} & \frac{\partial Q_i}{\partial V_k} \\
\frac{\partial P_j}{\partial \theta se_{ji}} & \frac{\partial P_j}{\partial Vse_{ji}} & 0 & 0 & \frac{\partial P_j}{\partial \theta_i} & \frac{\partial P_j}{\partial V_i} & \frac{\partial P_j}{\partial \theta_j} & \frac{\partial P_j}{\partial V_j} & 0 & 0 \\
\frac{\partial Q_j}{\partial \theta se_{ji}} & \frac{\partial Q_j}{\partial Vse_{ji}} & 0 & 0 & \frac{\partial Q_j}{\partial \theta_i} & \frac{\partial Q_j}{\partial V_i} & \frac{\partial Q_j}{\partial \theta_j} & \frac{\partial Q_j}{\partial V_j} & 0 & 0 \\
0 & 0 & \frac{\partial P_k}{\partial \theta se_{ki}} & \frac{\partial P_k}{\partial Vse_{ki}} & \frac{\partial P_k}{\partial \theta_i} & \frac{\partial P_k}{\partial V_i} & 0 & 0 & \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial V_k} \\
0 & 0 & \frac{\partial Q_k}{\partial \theta se_{ki}} & \frac{\partial Q_k}{\partial Vse_{ki}} & \frac{\partial Q_k}{\partial \theta_i} & \frac{\partial Q_k}{\partial V_i} & 0 & 0 & \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial V_k}
\end{bmatrix}
\times
\begin{bmatrix} \Delta \theta se_{ji} \\ \Delta Vse_{ji} \\ \Delta \theta se_{ki} \\ \Delta Vse_{ki} \\ \Delta \theta_i \\ \Delta V_i \\ \Delta \theta_j \\ \Delta V_j \\ \Delta \theta_k \\ \Delta V_k \end{bmatrix}
\tag{91}
$$

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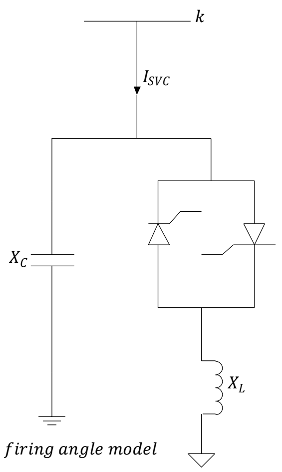
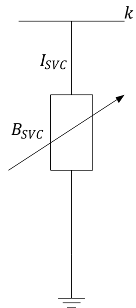
H. INCORPORATION OFSTATIC VAR COMPENSATOR (SVC)INTO THE LOAD FLOW EQUATIONS

Static VAR Compensator (SVC) is ashunt compensatorFACTS controller based on thyristor controlled reactor (TCRs). It is capable of regulating terminal voltages either by injecting reactive power when the system voltage is low or absorbing reactive power from the system when system voltage is high [25, 27].Theeffectiveness of SVC depends on itsoptimal location and proper signal selection in thepower system as it can be used to improve both voltage and reactive power conditions of the system[27] Also, it can as well improve system stability and damping by dynamicallycontrolling its reactive power output.
The SVC firing angle and total susceptance modelis shown below in fig.8 and the details of thepower flow equations incorporating SVC with Newton Raphson iterativealgorithm is reported in [25, 28].



$Fig.8a: SVC\ firing\ angle\ model$



$Fig.8b: SVC\ total\ susceptance$

Figure 8: shows the firing angle and total susceptance model of SVC [25]
The SVC effective reactance X_{SVC} is determined by the parallel combination of X_C and X_{TCR} and is given by;

$$X_{SVC} = \frac{\pi X_C X_L}{X_C[2(\pi-\alpha)+\sin 2\alpha]-\pi X_L} \qquad (92)$$

With SVC incorporation to improve defective bus voltage, the modified Jacobian matrix is given by the equation below;

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$$\begin{bmatrix} \Delta P_I \\ \Delta P_k \\ \Delta P_n \\ \Delta Q_I \\ \Delta Q_k \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_I}{\partial \delta_I} & \frac{\partial P_I}{\partial \delta_k} & \frac{\partial P_I}{\partial \delta_n} & \frac{\partial P_I}{\partial V_I} & 0 & \frac{\partial P_I}{\partial V_n} \\ \frac{\partial P_k}{\partial \delta_I} & \frac{\partial P_k}{\partial \delta_k} & \frac{\partial P_k}{\partial \delta_n} & \frac{\partial P_k}{\partial V_I} & 0 & \frac{\partial P_k}{\partial V_n} \\ \frac{\partial P_n}{\partial \delta_I} & \frac{\partial P_n}{\partial \delta_k} & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_I} & 0 & \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_I}{\partial \delta_I} & \frac{\partial Q_I}{\partial \delta_k} & \frac{\partial Q_I}{\partial \delta_n} & \frac{\partial Q_I}{\partial V_I} & 0 & \frac{\partial Q_I}{\partial V_n} \\ \frac{\partial Q_k}{\partial \delta_I} & \frac{\partial Q_k}{\partial \delta_k} & \frac{\partial Q_k}{\partial \delta_n} & \frac{\partial Q_k}{\partial V_I} & 0 & \frac{\partial Q_k}{\partial V_n} \\ \frac{\partial Q_n}{\partial \delta_I} & \frac{\partial Q_n}{\partial \delta_k} & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_I} & 0 & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \begin{bmatrix} \Delta \partial_I \\ \Delta \partial_k \\ \Delta \partial_n \\ \Delta V_I \\ \Delta \alpha \\ \Delta V_n \end{bmatrix} \quad (93)$$

If $Q_k = Q_k^{old} + Q_{SVC}$ (94)

Then;

$$\frac{\delta Q_k}{\delta \alpha} = \frac{\delta Q_k^{old}}{\delta \alpha} + \frac{\delta Q_{SVC}}{\delta \alpha} \quad (95)$$

Since Q_k depends on the firing angle (α) then,

$$\frac{\delta Q_k}{\delta \alpha} = \frac{\delta Q_{SVC}}{\delta \alpha} \quad (96)$$

With $Q_{SVC} = -y_{SVC} V_k^2$then;

$$Q_{SVC} = -\frac{V_k^2}{X_L X_C}\left[X_L - \frac{X_C}{\pi}(2(\pi - \alpha) + \sin 2\alpha)\right] \quad (97)$$

Therefore equation (83) is reduced to;

$$\frac{\delta Q_k}{\delta \alpha} = \frac{2V_k^2}{\pi X_L}(\cos(2\alpha) - 1) \quad (98)$$

III. CONCLUSION

An Overview of MathematicalSteady-StateModelling of Newton-RaphsonLoad Flow Equations Incorporating LTCT, Shunt Capacitor and FACTSDeviceshas been carried out. Newton-Raphson iterative technique was adopted in the modelling due to its superior advantagesover other iterative techniques such as better reliability since it converges faster with quadratic convergencecharacteristics and with the least number of iterations aside being independent of the system size.The modelling shows that incorporation of both discrete and FACTS controllers increase the robustness and versatility of Newton-Raphson solution methodology of power flow analysis.

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