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Analysis of Quantised Digital FIR Filter Using Different Windowing Techniques

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ABSTRACT: Digital filters have a wide application in digital signal processing. This paper gives a brief introduction to the design of Finite Impulse Response (FIR) digital filters using Windowing method. A window is a finite array consisting of coefficients selected to satisfy the requirements. This paper concerns with the design and implementation of FIR filters using a various window method like Hann, hamming, Kaiser, Tukey, Blackman and Rectangular.

KEYWORDS: FIR filters, window function, FDA tool in MATLAB

I. INTRODUCTION

A filter plays a very important role in signal processing. In signal processing there may be cases where an input signal may contain unwanted noise which can produce undesired effect. The objective of filtering is to perform signal separation or signal restoration and eventually improve the quality of the signal. A digital filter is far more superior in level of performance than analog filter and thus plays a very important role in Digital Signal Processing [18].

II. BASIC PRINCIPLE OF FIR FILTER

There are two types of filters basically.

- 1) Finite Impulse response (FIR)
- 2) Infinite impulse response (IIR)

FIR filters are Non-Recursive in nature which means there is no feedback involved, IIR filters are Recursive which means that feedback is present. If phase characteristic is of essence then FIR filters should be used, such filters have high order and are very complex. But, if only frequency response matters then IIR filters with low order and less complexity may be used.

In 2007 Mishra Amit and Krishna Raj presented the idea for "Implementation of a digital neuron with nonlinear activation function using piecewise linear approximation technique" [1]. The concept of "Design of Efficient Digital Filter Using Cut-set Reimimg Technique" was described by Vishal Awasthi and Krishna Raj [2]. In order to design a pipelined IIR filter a tutorial using universal look-ahead algorithm was presented by Krishna Raj and Vivekanand yadav [3]. For the study of genetic algorithm for IIR adaptive filters an approach was described by Krishna Raj, Subhash Chandra Yadav and R.G. Vashne [4]. For the purpose of designing and implementing efficient algorithms and concept of digital filters researchers have described Kalman Filter, CIC compensated filter, modified hybrid sign digit concept, cordic algorithm [5-15]

Figure 1 shows the Magnitude response of physically realizable filter. In an ideal environment magnitude may be constant in the pass band of a filter but in practical approach a small amount of ripple can be tolerated in the pass band as well in the stop band. ω_p represents the pass band edge and ω_s represents the stop-band edge. $(\omega_s - \omega_p)$ defines the width of the transition band [19]. Bandwidth of the filter is defined by pass-band width. Pass-band ripple is represented by δ_1 and if there is ripple in the stop-band it is denoted by δ_2 . When designing a FIR filter it is necessary to specify

pass-band, stop-band, transition-band. In pass-band frequency must be passed unattenuated and in stop-band frequency must be passed attenuated.

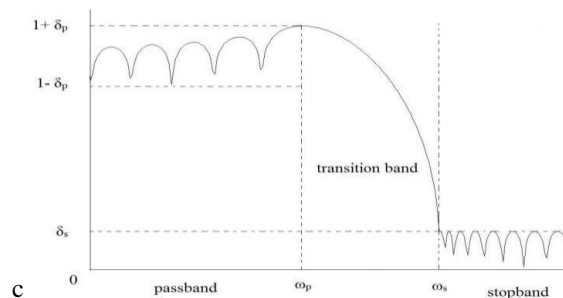


Fig.1 Magnitude response of physically realizable Filter

The basic hardware requirement for designing a FIR filter is multipliers, delay elements and adders. Figure 2 shows the structure of FIR Filter of order N.

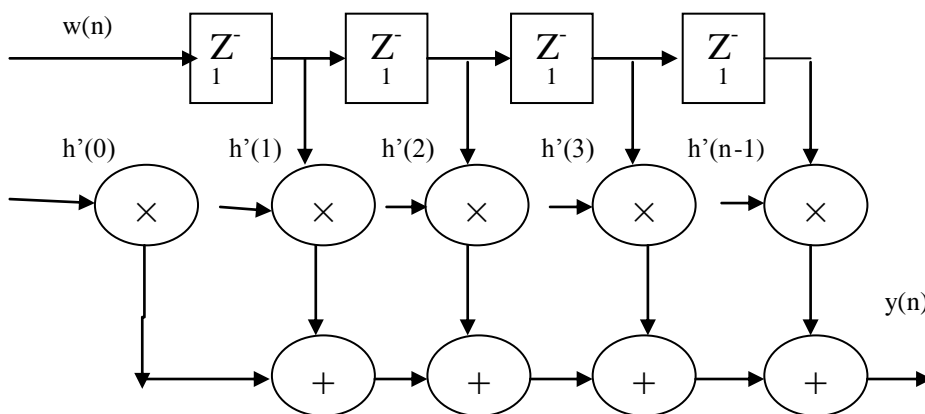


Fig.2 Structure of FIR Filter of order N

The input and the output signals for the IIR filter:

$$W(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \rightarrow (1)$$

From the above equation we see that the impulse response of IIR Filter is of infinite duration, therefore the output of IIR filter using equation (1) is practically unachievable.

Equation (1) can be also be expressed in a recursive form

$$W(n) = \sum_{k=0}^M b_k(n-k) - \sum_{k=1}^N a_k y(n-k) \rightarrow (2)$$

Where a_k and b_k are the filter coefficients

The difference equation of an N order FIR filter is:

$$W(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \rightarrow (3)$$



Where:

$x(n)$ is the input signal to the filter

$h(k)$ is the impulse response

K is the order of filter

$W(n)$ is the output of FIR filters

From the equations (3) we can deduce that the impulse response of FIR filter is of finite duration.

For FIR Filter, the transfer function can be obtained by:

$$H(z) = \sum_{k=0}^{N-1} h(k) z^{-k} \rightarrow (4)$$

For IIR Filter, the transfer function can be obtained by:

$$H(z) = \frac{\sum_{k=0}^M b_k z^{-k}}{1 + \sum_{k=1}^N a_k z^{-k}} \rightarrow (5)$$

III. DESIGN METHODS FOR FIR FILTER

Most of the FIR filter design methods are based on the approximation of an ideal filter. The more the designed filter is close to the ideal filter the more will be its order, thus making the filter and its implementation more complex. The designer should try to achieve a compromise between the filter characteristics and order.

Filter design starts with specifications and requirements of the desirable FIR filter. The designer gets to choose the method based on the filter specifications. Each method has its own set of advantages and disadvantages. Choosing the right method is very crucial in any FIR filter design process

There are essentially three well-known methods for FIR filter design namely:

- 1) The window method
- 2) The frequency sampling technique
- 3) Optimal filter design methods

Throughout this paper we have discussed the window method for FIR filter design. The simplicity of design process makes this method very popular.

IV. THE WINDOW METHOD

A window is defined as a finite array of coefficients that will meet the requirements. There are basically two kinds of windows: fixed window function and adjustable window function. Hanning, Hamming, Blackman and rectangular windows are fixed window function. Kaiser, Tukey window are adjustable window function.[16]

When designing digital FIR filters using window functions it is necessary to specify:

- 4) A window function that will be used; and
- 5) The filter order;

The filter parameters used in the current study are as follows:

Filter order=10

Sampling Frequency=48000Hz

Cut-off Frequency=10800Hz

Filter Type=Low-Pass

Hanning window: The Hanning window is also known as a raised cosine window [10]. The advantage of using Hanning Window method is that it reduces the side lobes. It has good frequency resolution over other window method like rectangular window. The Hanning Window function can be mathematically defined as:

$$W = \begin{cases} 0.5 \left(1 - \frac{\cos 2\pi n}{M-1} \right) \\ 0 \end{cases}, \text{from } = 0 \text{ to } n-1$$

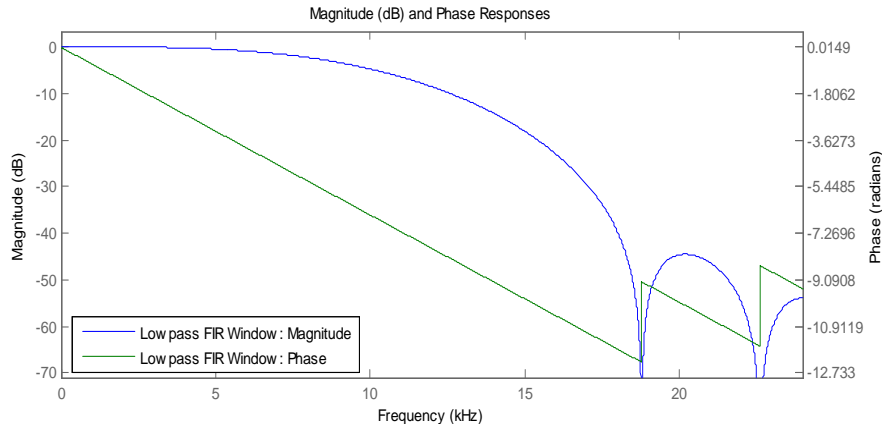


Fig. 3 Magnitude and phase response of Hanning window

Hamming window The Hamming window is also known as a raised cosine window. Hamming window exhibit characteristic similar to the Hanning window. It suppresses the side lobe. [10] Mathematically, the Hamming window function can be written as:

$$W(n) = \begin{cases} 0.54046 \cos\left(\frac{2\pi n}{N}\right), & \text{from } 0 \text{ to } n-1 \\ 0, & \text{elsewhere} \end{cases}$$

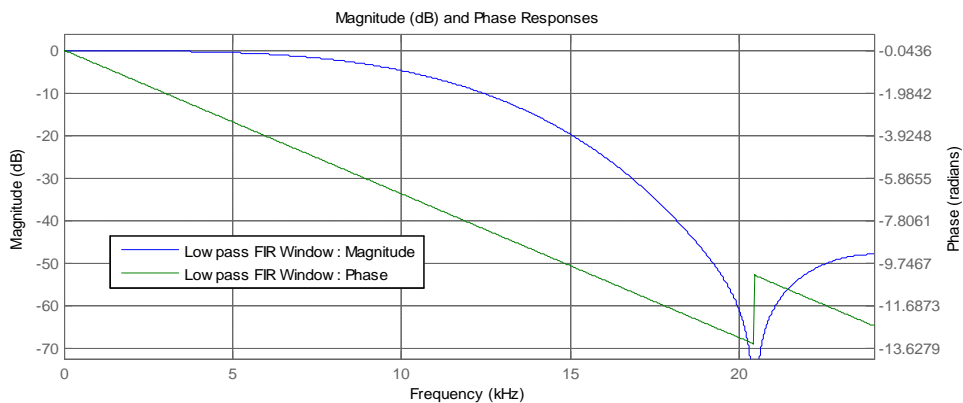


Fig. 4 Magnitude and phase response of Hamming window

Blackman window: If we compare Blackman window with other windows there is better improvement in stop band attenuation [10]. The Blackman window function can be written as:

$$W(n) = \begin{cases} 0.42 - 0.5 \cos\left(\frac{2\pi n}{N}\right) + 0.08 \cos\left(\frac{4\pi n}{N}\right), \\ 0, \end{cases} \begin{matrix} \text{from } 0 \text{ to } n-1 \\ \text{elsewhere} \end{matrix}$$

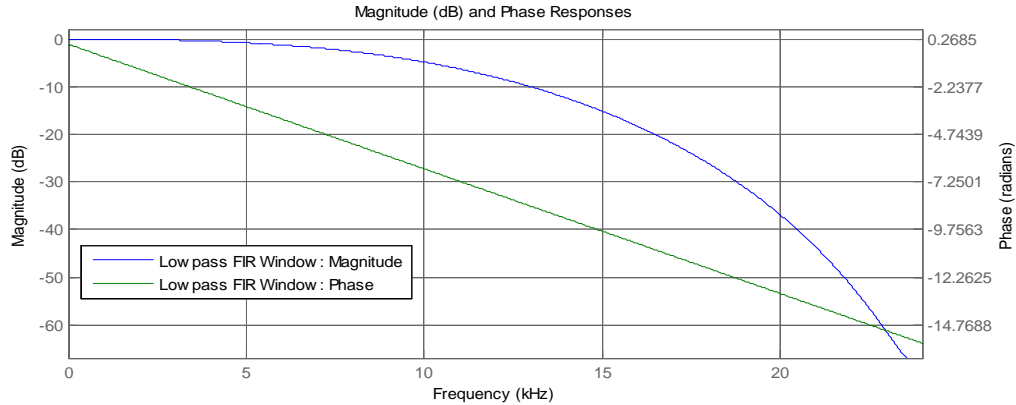


Fig. 5 Magnitude and phase response of Blackman window

Kaiser window: As compared to the other window functions the Kaiser Window has maximum attenuation according to the given width of the main lobe. That is why it is also known as optimal window. The Kaiser Window function can be written:

$$W(n) = \frac{I_0\left(\beta \sqrt{1 - \left(\frac{n-\alpha}{\alpha}\right)^2}\right)}{I_0(\beta)} \text{ from } 0 \text{ to } n-1$$

$$\alpha = \frac{N-1}{2}$$

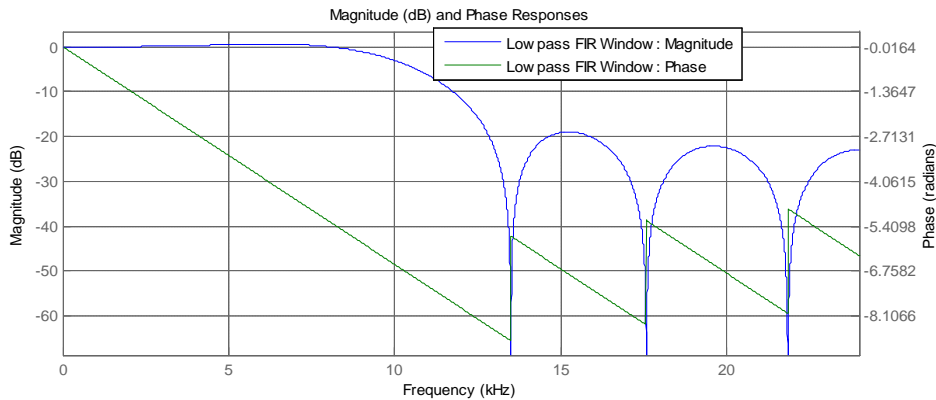


Fig. 6 Magnitude and phase response of Blackman window

Tukey Window The Tukey window is also called the tapered cosine window. The window function for the Tukey Window can be defined as follows:

$$W_r(n) = \begin{cases} \frac{1}{2} \left[1 + \cos \left(\pi \left(\frac{2n}{\alpha(M-1)} - 1 \right) \right) \right], & 0 \leq n \leq \frac{\alpha(M-1)}{2} \\ 1, & \frac{\alpha(M-1)}{2} \leq n \leq (M-1) \left(1 - \frac{\alpha}{2} \right) \\ \frac{1}{2} \left[1 + \cos \left(\pi \left(\frac{2n}{\alpha(M-1)} - \frac{2}{\alpha} + 1 \right) \right) \right], & (M-1) \left(1 - \frac{\alpha}{2} \right) \leq n \leq (M-1) \end{cases}$$

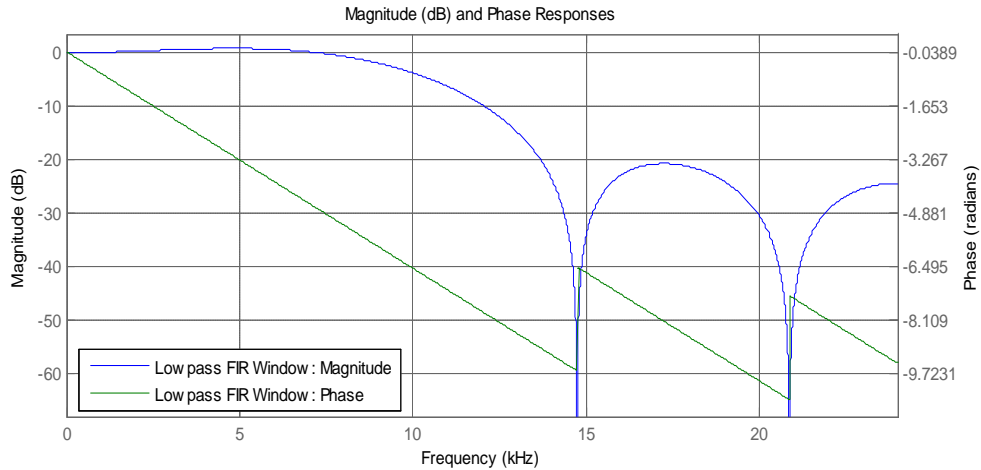


Fig. 7 Magnitude and phase response of Tukey window

Rectangular Window Function The weighting function for the Rectangular window is given by easily and quickly. The rectangular window is rarely used for its low stopband attenuation. The first lobe as attenuation of 13dB and the narrowest transition region, therefore. A filter designed using this window has minimum stopband attenuation of 21 dB.

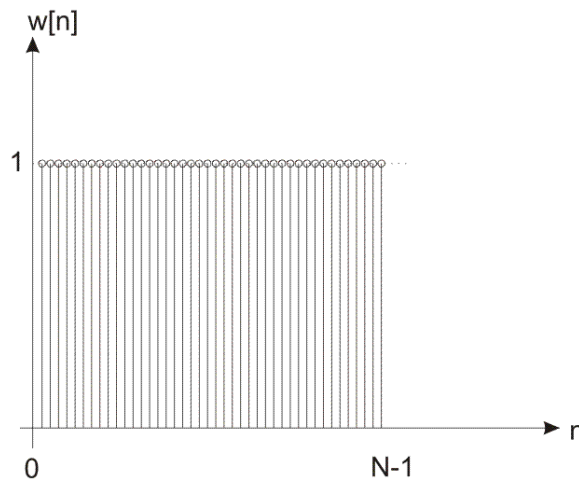


Fig. 8 Rectangular window in time domain

Unlike other window functions being a kind of compromise between requirements for as narrow transition region as possible and as high stop band attenuation as possible, this window is characterized by extreme values. Namely, the minimum transition region is achieved, but at a cost of stop band attenuation.

It is easy to find rectangular window coefficients as all coefficients between 0 and N-1 (N-filter order) are equal to 1, which can be expressed in the following way:

The window function for rectangular Window can be defined by the function:

$$W_R(n) = \begin{cases} 1, & \text{for } n \leq \frac{M-1}{2} \\ 0, & \text{elsewhere} \end{cases}$$

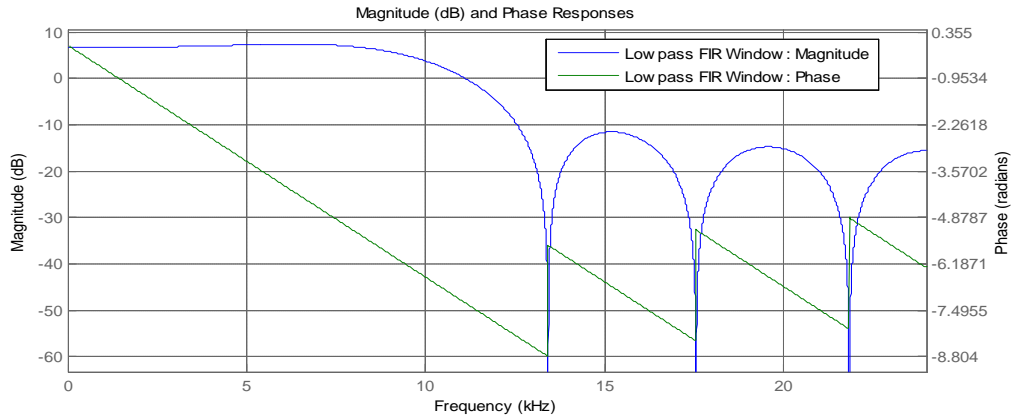


Fig. 9 Magnitude and phase response of Rectangular window

The characteristics of the window play a significant role in determining the resultant frequency response of the FIR filter that is obtained by truncating the impulse response $h_d(n)$ to a pre-decided length M . The convolution of $H_d(\omega)$ with $W(\omega)$ has the effect of smoothing of $H_d(\omega)$.

Where:

$h_d(n)$ is the unit sample response

M is the length of the filter

$H_d(\omega)$ is the desired frequency response specification

$W(\omega)$ is the frequency domain representation of the window function

The frequency response characteristic of the Hamming and Blackmann windows are illustrated in the figs: fig 4 through fig 5. All these functions have significantly lower side bands compared to rectangular window but for the same value of M the width of the main lobe is also wider for these windows compared to the rectangular window. Hence these window function provide more smoothing through the convolution operation in the frequency domain and as a result, the transition region is wider. To overcome this problem we can increase the length of the window but it results in larger filter. Comparison of these window functions is summarised in the table below.

Table1. Properties of window function

Window used	Main lobe width Δ_{ML}	Relative sidelobe level A_{sl}	Minimum stopband attenuation	Transition bandwidth $\Delta\omega$
Hann	$8\pi/(2M+1)$	31.5 Db	43.9 dB	$3.11\pi/M$
Hamming	$8\pi/(2M+1)$	42.7 Db	54.5 dB	$3.32\pi/M$
Blackman	$12\pi/(2M+1)$	58.1 dB	75.3 dB	$5.56\pi/M$
Rectangular	$4\pi/(2M+1)$	13.3 Db	20.9 dB	$0.92\pi/M$

V.FIR FILTER DESIGN USING FDA TOOL

FDATool in the MATLAB signal processing toolbox is used for designing of filter. It is very simple and convenient to use. The tool takes in a set of parameters required for filter designing and gives the resulted designed filter which then can be analysed to meet the specifications. The first step in any filter design process is to design the filter with double floating point and then truncate the filter coefficients to implement the fixed-point hardware. This is done because most of the DSP systems used fixed-point arithmetic. In real time it may be possible that the quantised filter does not fit the requirements, in such case we need to backtrack and adjust the parameters to fit the requirements.

The filter parameters used in the current study are as follows;

Filter order=10

Sampling Frequency=48000Hz

Cut-off Frequency=10800Hz
Filter Type=Low-Pass

Design of FIR Filter using Hanning window: fig.9 gives a comparative analysis of the reference and quantized filter coefficients.

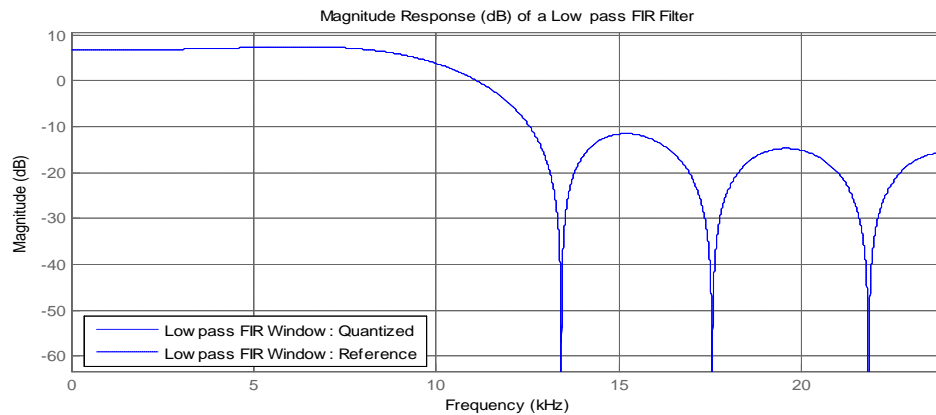


Fig. 10 Magnitude response of Hanning window

Hamming window: Fig.11 gives a comparative analysis of the reference and quantized filter coefficients.

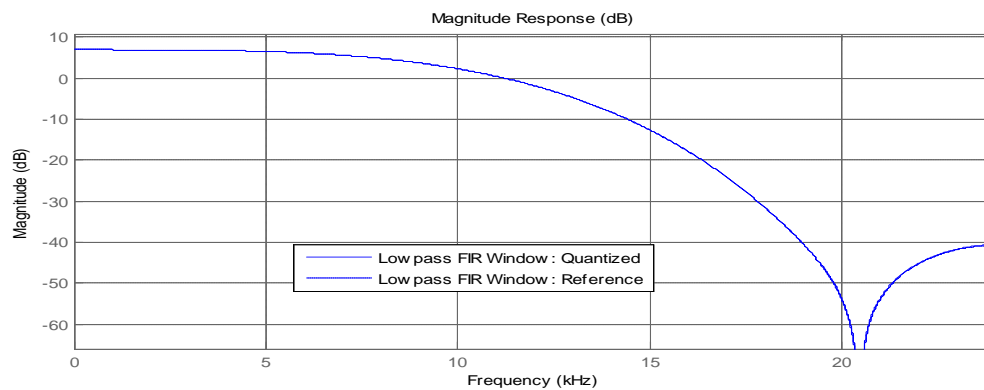


Fig. 11 Magnitude response of Hamming window

Blackman window: Fig.12 gives a comparative analysis of the reference and quantized filter coefficients.

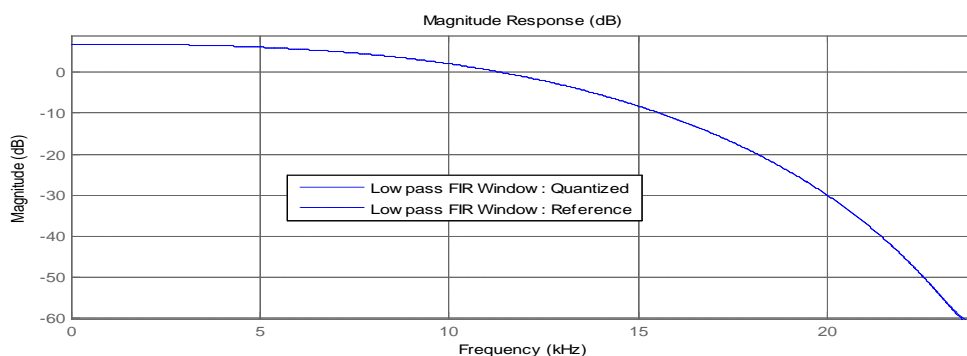


Fig. 12 Magnitude response of Blackman window

Kaiser window: Fig.13 gives a comparative analysis of the reference and quantized filter coefficients.

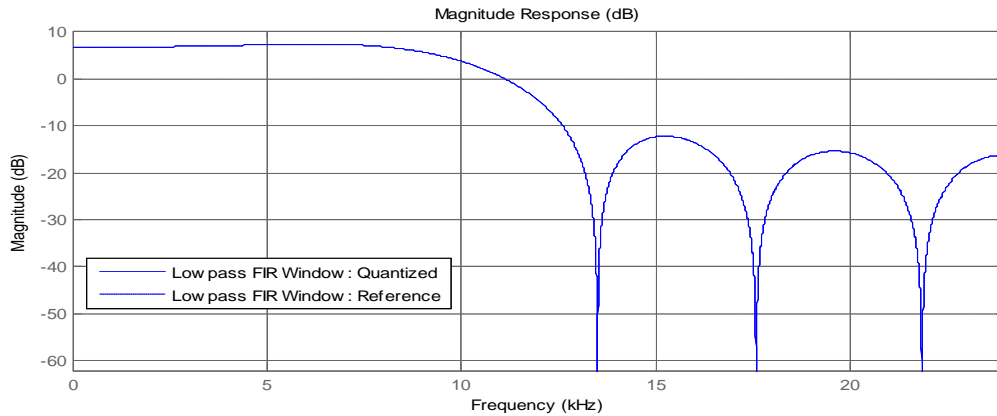


Fig. 13 Magnitude response of Kaiser window

Tukey window: Fig.14 gives a comparative analysis of the reference and quantized filter coefficients.

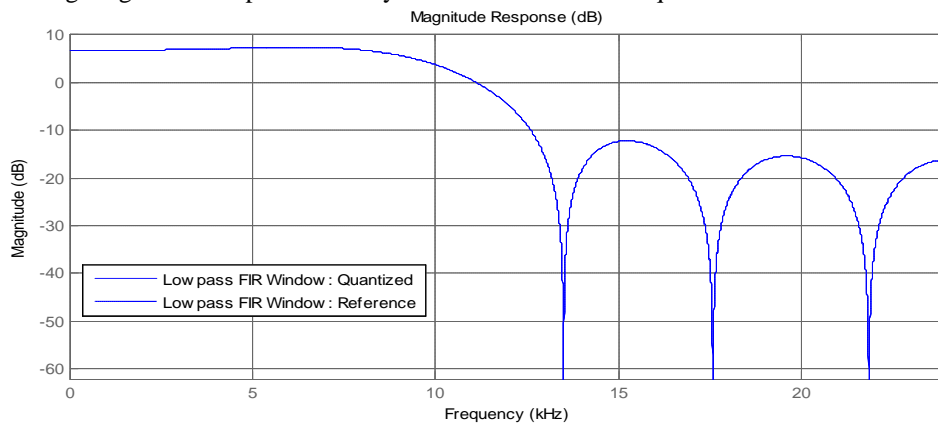


Fig. 14 Magnitude response of Tukey window

Rectangular window: Fig.15 gives a comparative analysis of the reference and quantized filter coefficients.

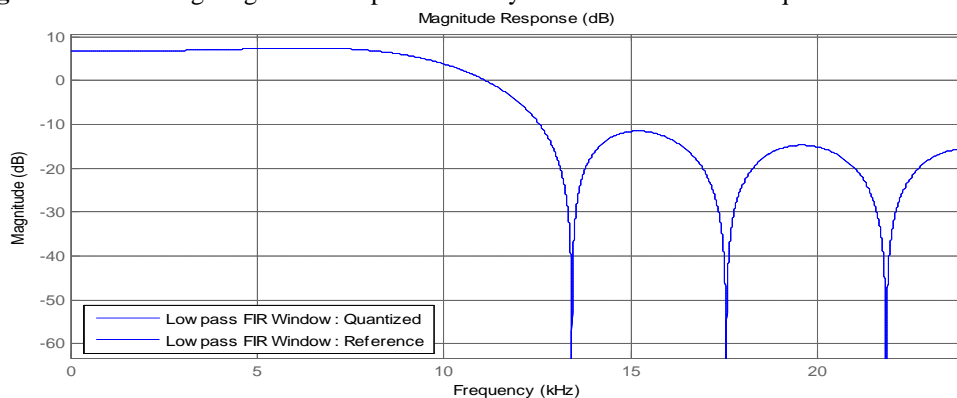


Fig. 15 Magnitude response of Rectangular window

The magnitude of the frequency response of low pass FI filter using rectangular window is fig.14 for M. It is observed that relatively large oscillations or ripples occur near the band of the filter. These oscillations increase in frequency as M increases but they do not diminish in amplitude. These oscillations near the band of the filter which are called the gibbs phenomenon contain decays in the passband as well in the stop band. So we should use a window function that

contains a taper and decay towards zero gradually and for this purpose we have used Hamming, Hann and Blackman windows.

Table2. Comparative Analysis of quantised filter using window method

Design Analysis of Discrete Time FIR Filter					
Window used	Filter Structure	Filter length	Stability?	Linear Phase	Arithmetic Used
Hann	Direct-Form FIR	11	Yes	Yes(Type-1)	Fixed
Hamming	Direct-Form FIR	11	Yes	Yes(Type-1)	Fixed
Blackman	Direct-Form FIR	11	Yes	Yes(Type-1)	Fixed
Kaiser	Direct-Form FIR	11	Yes	Yes(Type-1)	Fixed
Tukey	Direct-Form FIR	11	Yes	Yes(Type-1)	Fixed
Rectangular	Direct-Form FIR	11	Yes	Yes(Type-1)	Fixed

IMPLEMENTATION COST OF DESCRETE TIME FIR FILTER					
Window used	No. of Multipliers	No. of Adders	No. of States	Multiplications per I/P Sample	Additions per I/P Sample
Hann	8	8	10	8	8
Hamming	10	10	10	10	10
Blackman	10	10	10	10	10
Kaiser	10	10	10	10	10
Tukey	8	8	10	8	8
Rectangular	10	10	10	10	10

VI. CONCLUSION

In Digital Signal processing digital filters are more preferable than analog filters, because they can be easily designed and can be used in various types of signal filtering applications. The designer has the flexibility to choose the technique for designing the filter. This paper discuss the Window method and does the comparison between the various window functions The major advantages of Window technique is its simplicity and easy to understand. Using FDA tool in Mat lab, we can design Digital filter very easily to suit the specifications. As per the analysis an ideal window is a delta function $w(\omega) = \delta(\omega)$ which required the rectangular function given in the equation

$$w(i) = \begin{cases} 1 & i = 0, 1, 2 \dots \dots i - 1 \\ 0 & \text{otherwise} \end{cases}$$

to have an infinite length otherwise $w(\omega)$ will have the classic sinc(x) shape. The rippling and wide transition regions of FIR filters are generated by this convolution. Thus the most effective method for reducing these effects is to design a finite length window such that the resulting impulse response decays towards zero smoothly instead of using the rectangular window defined in the above equation. The best window for most of the applications is the Kaiser Window which has parameters to adjust the compromise between the overshoot reduction and transition band.



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