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Common fixed point theorems for a weak ** commuting pair of mappings

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ABSTRACT: The concept of weak* commuting mappings was given by H.K. Pathak [3]. has generalized some results of B. Fisher [2] on fixed point theorem by using the concept to weak ** commuting mapping. We have two common fixed point theorems for three self maps of a complete metric space satisfying a rational inequality by using the concepts of weak ** commuting maps and rotativity of maps. We further extend the results of Diviccaro, Sessa and Fisher [1].

KEYWORDS: Weak ** commuting, Idempotent, Rotative, Complete metric space,

Some Definitions.

We begin with the following known definitions:-

Definition 1: Let (X,d) be a space and let S and I be mappings of X in to itself. We define the pair (S,I) to be weak ** commuting.

if $S(X)\subset I(X)$

and $d(S^2I^2x, I^2S^2X) \leq d(S^2Ix, IS^2x) \leq d(SI^2x, I^2Sx) \leq d(SIx, ISx) \leq d(S^2x, I^2)$

for all x in X.

It is obvious that two commuting mapping are also weak ** commuting, but two weak**commuting do not necessarily commute as shown in exampole 1 below.

Definition 2 : A map $T:X\rightarrow X$ is called idempotent, if $T^2 = T$. We note that if mappings are idempotent, then our definition of weak ** commuting of pair (S,I) reduces to weak commuting of pair (S,I) defined by Sessa [5].

Definition 3 : The map T is called rotative w.r.t.I, If $d(Tx, I^2x) \leq d(Ix, T^2x)$

for all x in X. clearly if T and I are idempotent maps, then definition is obvious.

Common fixed point theorems for a weak ** commuting pair of mappings.

In this section, we have some results on common fixed points for three self maps of a complete metric space satisfying a rational inequality by using the concepts of weak ** commuting maps and rotativity of maps. The following theorem generalizes the result of Diviccaro, Sessa and fisher [1]

Theorem 1 Let S, T and I be three mappings of a complete metric space (X,d) such that foa all x, y in X either

(I) $d(S^2x, T^2y) \leq K' [d(I^2x, S^2x) + d(I^2y, T^2y)]$

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$$+K\frac{[d(I^2x,S^2x).d\,(I^2y,T^2y)+\ d\,(I^2x,T^2y).d\,(I^2y,S^2x)]}{d(I^2\,x,S^2x)+d\,(I^2y,T^2y)}$$

if $d\,(I^2x, S^2x)+ d\,(I^2y, T^2y) \neq 0$, where $K' < 1$, and $(K+K')<1/2$, or

(II) $d(S^2x, T^2y) = 0$ if $d\,(I^2x, S^2x)+ d\,(I^2y, T^2y)=0$

Suppose that the range of I^2 contains the range of S^2 and T^2. If either

(a_1) I^2 is continuous, I is weak ** commuting with S and T is rotative w.r.t.I,

(a_2) I^2 is continuous, I is weak ** commuting with T and S is rotative w.r.t.I,

(a_3) S^2 is continuous, S is weak ** commuting with I and T is rotative w.r.t.S,

(a_4) T^2 is continuous, T is weak ** commuting with I and S is rotative w.r.t.T

Then S, T and I have a unique common fixed point z. Further, z is the unique common fixed point of S and I and T and I.

Inspired by the result of Pathak H.K. and Sharma, Rekha [6], in the next Theorem, we generalize the Theorem of Rathore, M.S. and Dolas, Uday [4].

But firstly this definition follows:

Let R^+ be the set of non-negative real numbers and N be the set of positive integers. Let $\Psi : R^+ \rightarrow R^+$ be a continuous and increasing function on R^+ such that

$\Psi(t) = 0$ it and only it $t = 0$.

Theorem 2.

Let S, T and I be the three self mappings of a complete metric space (X,d) satisfying the following condition:

(IV) $\Psi\,(d(T^2x,S^2y)) \leq A.\ \max.\{\Psi(d(I^2x,I^2y)), [1/2.\ \Psi(d(I^2x,I^2y))\ \Psi(d(I^2y,T^2x))]^{1/2}\}$

$+ B.\{\Psi(d(I^2x,T^2x)) + \Psi(d(I^2y, S^2y))\}$

$+ C.\min.\{\Psi(d(I^2x,S^2y)), \Psi(d(I^2y,T^2x))\}$

$\forall\ x,y \in X$ and reals $A,B,C \leq O$ with $(A + 2B+ C) < 1$.

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Suppose that the range of I^2 contains the range of S^2 and T^2. If either

(a$_1$) I^2 is continuous, I is weak ** commuting with S and T is rotative w.r.t.I,

or (a$_2$) I^2 is continuous, I is weak ** commuting with T and S is rotative w.r.t.I,

or (a$_3$) S^2 is continuous, S is weak * * commuting with I and T is rotative w.r.t.S,

or (a$_4$) T^2 is continuous, T is weak * * commuting with I and S is rotative w.r.t.T,

Then S,T and I have a common fixed point z, further z is a unique common fixed point of the pairs {S,I} , {T,I} and {S,T}.

Proof: Let x_o be an arbitrary point in X. Since the range of I^2 contains the range of S^2. Let x_1 be a point in X such that $S^2x_0=I^2x_1$. Since the range of I^2 contains the range of T^2, we can choose a point x_2 such that $T^2x_1 = I^2x_2$.

In general we have

$S^2x_{2n} = I^2x_{2n+1}$ and $T^2x_{2n+1} =I^2x_{2n+2}$ for n = 0,1,2 ....

Put $d_{2n-1} = d(T^2x_{2n-1},S^2x_{2n})$ and $d_{2n} = d(S^2x_{2n,}T^2x_{2n+1})$ for n = 1, 2...........

Now we distinguish three cases.

Case I. Let $d_{2n-1}\neq 0$ and $d_{2n} \neq 0$, for n = 1,2 ....

Using inequality (IV), we have

$\Psi(d(T^2X_{2n+1,}\ S^2x_{2n}))$

$\leq$ A. max { $\Psi(d(I^2x_{2n+1},I^2\ x_{2n}))$, $[1/2.\ \Psi(d(I^2x_{2n+1,}I^2x_{2n})).\ \Psi(d(I^2x_{2n,}T^2x_{2n+1}))]^{1/2}$}

+ B. {$\Psi(d(I^2x_{2n+1,}T^2x_{2n+1})) + \Psi(d(I^2x_{2n'}S^2x_{2n}))$}

+ C. min{$\Psi(d(I^2x_{2n+1'}S^2x_{2n}))$, $\Psi(d(I^2x_{2n'}\ T^2x_{2n+1}))$}

i.e. $\Psi(d_{2n}) \leq$ A. max. {$\Psi(d_{2n-1})$, $[1/2\Psi(d_{2n-1}).\ \Psi\ (d_{2n-1}+d_{2n})]^{1/2}$}

+ B. {$\Psi(d_{2n}) + \Psi(d_{2n-1})$} + C.min. {$\Psi(0)$, $\Psi(d_{2n-1}+ d_{2n})$}

i.e. $\Psi(d_{2n}) \leq$ A. max. {$\Psi(d_{2n-1})$, $[1/2.\Psi(d_{2n-1}).\Psi(d_{2n-1}+d_{2n})]^{1/2}$}

+ B. {$\Psi(d_{2n}) + \Psi(d_{2n-1})$}.

Suppose that $.\Psi(d_{2n-1}) < (d_{2n})$.

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Then we have

$\Psi(d_{2n}) \leq A.\max\{\Psi(d_{2n}), [1/2.\Psi(d_{2n}).(\Psi(d_{2n}) + \Psi(d_{2n}))]^{1/2}\}$

$+ B.\{\Psi(d_{2n}) + \Psi(d_{2n})\}$

So that $\Psi(d_{2n}) \leq A.\Psi(d_{2n}) + 2B.\Psi(d_{2n})$.

i.e. $\Psi(d_{2n}) \leq (A+2B).\Psi(d_{2n})$.

Therefore $\Psi(d_{2n}) < \Psi(d_{2n})$., since $(A+2B) < 1$.

So that our assumption is wrong, then we have

$\Psi(d_{2n}) \leq \Psi(d_{2n-1})$.

Simirlarly we have

$\Psi(d_{2n-1}) \leq \Psi(d_{2n-2})$.

Thus we have $\Psi(d_{n-1}) \leq (d_n)$, $\forall n = 1,2$ ....

Since Ψ is an increasing function, we conclude that $\{d_n\}$ is a decreasing sequence of non-negative real numbers.

Thus $d_{2n} \leq d_{2n-1} \leq d_{2n-2} \leq$.............. $\forall n = 1,2$ ....

It follows that the sequences

(4) $\{S^2x_0, T^2x_1, S^2x_2, \ldots\ldots\ldots T^2x_{2n-1}, S^2x_{2n}, T^2x_{2n+1}\ldots\ldots\}$

is a Cauchy sequence in the complete metric space X and so has a limit w in X, Hence the sub- sequences

$\{S^2x_{2n}\} = \{I^2x_{2n+1}\}$ and $\{T^2x_{2n-1}\}=\{I^2x_{2n}\}$

converge to the point we because they are subsequences of the sequence (4)

Suppose first of all that I^2 is continuous, then sequences $\{I^4x_{2n}\}$ and $\{I^2S^2x_{2n}\}$ converge to a point I^2w.

if I weak ** commutes with S, we have

$d(S^2I^2x_{2n},I^2w) \leq d(S^2I^2x_{2n},I^2S^2x_{2n}) + d(I^2S^2x_{2n},I^2w)$

$\leq d(S^2{}_{2n},I^2x_{2n}) + d(I^2S^2x_{2n},I^2w)$

which implies, on letting a tends to infinity that the sequence $\{S^2I^2x_{2n}\}$ also converge to I^2w.

We now claim that $T^2w = I^2w$. suppose not.

then we have $d(I^2w, T^2w) > 0$ using inequality (IV), we obtain

$\Psi(d(T^2w, S^2I^2x_{2n}))$

$\leq A.\max\{\Psi d(I^2w, I^4x_{2n})), [½.\Psi(d(I^2w, I^4x_{2n})). \Psi(d(I^4x_{2n,} T^2w,))]^{1/2}\}$

$+ B. \{\Psi(d(I^2w, T^2w,)+ \Psi(d(I^4x_{2n,} S^2I^2x_{2n}))\}$

$+C.\min\{\Psi(d(I^2w, S^2I^2x_{2n})), \Psi(d(I^4x_{2n,} T^2w))\}$

i.e. $\Psi(d(T^2w,I^2w))$

$\leq A.\max \{\Psi d(I^2w, I^2w)), [½.\Psi (d(I^2w, I^2w)). \Psi(d(I^2w, T^2w,))]^{1/2}\}$

$+ B. \{\Psi(d(I^2w, T^2w,)+ \Psi(d(I^2w, I^2w))\}$

$+C.\min\{\Psi(d(I^2w, I^2w)), \Psi(d(I^2w, T^2w))\}$

That is $\Psi(d(T^2w, I^2w) \leq B.\Psi(d(I^2w, T^2w))$.

Therefore $\Psi(d(T^2w, I^2w) \leq \Psi(d(I^2w, T^2w))$, Since B<1.

which is a contradiction and so our assumption is wrong. Hence have

$$T^2w = I^2w.$$

Now suppose that $S^2w \neq T^2w$. Then using inequality (IV), we have

$\Psi(d(T^2w, S^2w)) \leq A.\max\{\Psi d(I^2w, I^4w)), [½.\Psi(d(I^2w, I^2w)). \Psi(d(I^2w, T^2w,))]^{1/2}\}$

$+ B. \{\Psi(d(I^2w, T^2w)+ \Psi(d(I^2w, S^2w))\}$

$+ C.\min\{\Psi(d(I^2w, S^2w)), \Psi(d(I^2w, T^2w))\}$

i.e.

$\Psi(d(T^2w, S^2w)) \leq B.\Psi(d(T^2w, S^2w))$

Therefore $\Psi(d(T^2w, S^2w)) \leq \Psi(d(T^2w, S^2w))$, since B<1.

This is a contradiction and so our supposition is wrong and hence

$S^2w = T^2w$

Thus $I^2w = S^2w = T^2w$

A similar conclusion is achieved if I weak ** commutes with T.

Let us now suppose that S^2 is continuous instead of I^2. Then the sequences $\{S^4x_{2n}\}$ and $\{S^2I^2x_{2n}\}$ converse to the point S^2w. Now if S weak ** commutes with I, we have the sequence $\{I^2S^2x_{2n}\}$ also converges to S^2w.

Since the range of I^2 contains the range of S^2, there exist a point w',

such that $I^2w' = S^2w$.

Then if $T^2w' \neq S^2w = I^2w'$, we have by inequality (IV) we have

$\Psi(d(T^2w', S^4w_{2n})) = \Psi(d(T^2w', S^2S^2x_{2n}))$

$\leq A.\max\{\Psi d(I^2w, I^2S^2x_{2n})), [½.\Psi(d(I^2w, I^2S^2x_{2n})).\ \Psi(d(I^2S^2x_{2n}, T^2w'))]^{1/2}\}$

$+ B.\{\Psi(d(I^2w', T^2w') + \Psi(d(I^2S^2x_{2n}, S^4x_{2n}))\}$

$+C.\min\{\Psi(d(I^2w', S^4x_{2n})), \Psi(d(I^2S^2x_{2n}, T^2w'))\}$

i.e.

$\Psi(d(T^2w', S^2w)) \leq A.\max\{\Psi d(S^2w, S^2w)), [½.\Psi(d(S^2w, I^2w)).\ \Psi(d(S^2w, T^2w'))]^{1/2}\}$

$+ B.\{\Psi d(S^2w', T^2w')), +\Psi(d(S^2w, S^2w))\}$

$+ C.\min\{\Psi d(S^2w, S^2w)) + \Psi(d(S^2w, T^2w'))\}$

I.e. $\Psi(d(T^2w', S^2w)) \leq B.\Psi d(S^2w, T^2w'))$,

Therefore $\Psi(d(T^2w', S^2w)) \leq B.\Psi d(S^2w, T^2w'))$, since B < 1

Thus we arrive at a contradiction

Hence $S^2w = T^2w' = I^2w'$.

Now suppose that $S^2w \neq T^2w' = I^2w$. Then by inequality (IV), we have

$\Psi(d(T^2w', S^2w')) \leq A.\max\{\Psi d(I^2w', I^2w')), [½.\Psi(d(I^2w', I^2w')).\ \Psi(d(I^2w', T^2w'))]^{1/2}\}$

$+ B. \{\Psi(d(I^2w', T^2w',)+ \Psi(d(I^2w', S^2w'))\}$

$+C.min\{\Psi(d(I^2w', S^2w')), \Psi (d(I^2w', T^2w'))\}$

i.e.

$\Psi(d(T^2w', S^2w')) \leq B.\Psi(d(T^2w', S^2w'))$

Therefore $\Psi(d(T^2w', S^2w')) \leq \Psi(d(T^2w', S^2w'))$, since B<1.

This is a contradiction and so $I^2w' = S^2w' = T^2w'$.

A similar conclusion is obtained if one assume that T^2 is continuous and T is weak ** commuting with I.

Case II. Let $d_{2n-1} = 0$ for some n.

Then $I^2x_{2n} = T^2x_{2n-1}= S^2x_{2n} = I^2x_{2n+1}$ We claim $I^2x_{2n} = T^2x_{2n}$,

Otherwise $d(I^2x_{2n} , T^2x_{2n}) > 0$. By inequality (IV), we have

$0< \Psi(d(T^2x_{2n}, I^2x_{2n})= \Psi(d(T^2x_{2n}, S^2x_{2n}))$

$< A.max. \Psi(d(I^2x_{2n}, I^2x_{2n})),[½, \Psi(d(I^2x_{2n}, I^2x_{2n}). \Psi(d(I^2x_{2n}, T^2x_{2n}))]^{1/2})$

$+ B. \{\Psi(d(I^2x_{2n}, T^2x_{2n}) + \Psi(d(I^2x_{2n}, S^2x_{2n}))\}$

$+ C.min \{\Psi(d(I^2x_{2n}, S^2x_{2n}) + \Psi(d(I^2x_{2n}, T^2x_{2n}))\}$

i.e. $0 < B \Psi(d(T^2x_{2n}, I^2x_{2n}) \leq B. \Psi(d(I^2x_{2n}, T^2x_{2n})$

i.e. $(1-B) \Psi(d(T^2x_{2n}, I^2x_{2n}) \leq 0$.

This implies $I^2x_{2n}= T^2x_{2n} = S^2x_{2n}$

CaseIII. Let $d_{2n} = 0$ for some n.

Then $I^2x_{2n+1} = S^2x_{2n}= T^2x_{2n+1}$ We claim $I^2x_{2n+1} = S^2x_{2n+1}$,

Otherwise $d(I^2x_{2n+1} , S^2x_{2n+1}) > O$.

By inequality (IV), we have

$0< \Psi(d(I^2x_{2n+1}, S^2x_{2n+1}))= \Psi(d(T^2x_{2n+1}, S^2x_{2n-1}))$

$< A.max. \Psi(d(I^2x_{2n+1}, I^2x_{2n+1})),[½ \Psi(d(I^2x_{2n+1}, I^2x_{2n+1}). \Psi(d(I^2x_{2n+1}, T^2x_{2n+1})]^{1/2})$

$+ B. \{\Psi(d(I^2x_{2n+1}, T^2x_{2n+1}) + \Psi(d(I^2x_{2n+1}, S^2x_{2n+1}))\}$

$+ C.min \{\Psi(d(I^2x_{2n+1}, S^2x_{2n+1})), \Psi(d(I^2x_{2n+1}, T^2x_{2n+1}))\}$

i.e. $0< \Psi(d(I^2x_{2n+1}, S^2x_{2n+1})) \leq B. \Psi(d(I^2x_{2n+1}, S^2x_{2n-1}))$

i.e. $(1-B). \Psi(d(I^2x_{2n+1}, S^2x_{2n+1})) \leq 0.$

Since B < 1, we have

$I^2x_{2n+1} = S^2x_{2n+1} = T^2x_{2n+1}$

Thus we see that in all cases, there exists a point w such that

$I^2w = S^2w = T^2w = z$ (say).

Again if I weak ** commutes with S, we have

$d(S^2Iw, IS^2w) \leq d(SI^2w, I^2Sw) \leq d(SIw, ISw) \leq d(S^2w, I^2w) = 0$

which implies that

$S^2Iw= IS^2w, SI^2w =I^2Sw, SIw = ISw$ and so $I^2Sw = S^3w$.

Now we claim $Iz = z$. If not, then $IS^2w \neq T^2w$.

Therefore

$\Psi (d (IS^2w, T^2w)) = \Psi(d (T^2w, S^2w))$

$\leq A. max \{\Psi d(I^2w,I^3w)), [½.\Psi(d(I^2w,I^3w)). \Psi(d(I^3w,T^2w))]^{1/2}\}$

$+ B. \{\Psi d(I^2w, T^2w)),+\Psi(d(I^3w, S^2Iw))\}$

$+ C.min \{\Psi d(I^2w, S^2Iw)),\Psi(d(I^3w, T^2w))\}$

i.e. $\Psi(d(z,Iz)) \leq A. max \{\Psi d(z,Iz)), [½.\Psi (d(z, Iz)). \Psi (d(Iz, z'))]^{1/2}\}$

$+ B. \{\Psi d(z,z)),+\Psi(d(Iz, Iz))\}$

$+ C.\min\{\Psi d(z, Iz)),+\Psi(d(Iz, z))\}$

i.e. $\Psi d(z,Iz)) < (A+C). \Psi d(z,Iz))$

which is a contradiction, since (A+C) < 1

Hence $IS^2w = T^2W$ i.e. Iz = z Thus z is a fixed point of I.

Now we need to prove that $T^2z= z$ suppose $T^2z \neq z$. then we get

$\Psi (d(T^2z, z) = \Psi(d(T^2z, S^2w))$

$\leq A. \max\{\Psi d(I^2z, I^2w)), [½.\Psi(d(I^2z, I^2w)). \Psi(d(I^2w,T^2z))]^{1/2}\}$

$+ B. \{\Psi(d(I^2z, T^2z,)+ \Psi(d(I^2w, S^2w))\}$

$+ C.\min\{\Psi(d(I^2z, S^2w)), \Psi(d(I^2w, T^2w))\}$

i.e.

$\Psi(d(T^2z, z)) \leq A. \max\{\Psi d(z, z)),0\}, + B. [\Psi (d(z, T^2w))+0\}$

$+ C.\min\{\Psi (d(z, z)), \Psi (d(z, T^2z))\},$ Since $I^2z= z$

Thus, $\Psi (d(T^2z, z)) \leq B. \Psi (d(z, T^2z),$

Which is a contradiction, B < 1.

Therefore $T^2z = z$.

Now using the rotativity of T. w. r. t. I (or w.r.t. S) we have

$d(Tz, z)= d(Tz, I^2Z) \leq d(Iz, T^2z) = d(z,z) = 0$

Hence Tz = z, i, e, z is a fixed point of T.

Suppose $Sz \neq z$, then

$\Psi(d(Sz, z)) = \Psi(d(SI^2w, z)) = \Psi(d(I^2Sw,z))$

$= \Psi(d(S^3w, T^2w))$

$= \Psi(d(T^2w, S^2Sw))$

$\leq$A. max {Ψd(I^2w, I^2Sw)), [½.Ψ(d(I^2w, I^2Sw)). Ψ(d(I^2Sw,T^2w))]$^{1/2}$}

+ B. {Ψ(d(I^2w, T^2w)+ Ψ(d(I^2Sw, S^3w))}

+C.min{ Ψ (d(I^2w, S^3w)), Ψ(d(I^2Sw, T^2w))}

i.e.

Ψ(d(Sz, z)) $\leq$A. max {Ψd(z, Sz)), [½.Ψ(d(z, Sz)). Ψ(d(Sz, z))]$^{1/2}$}

+ B. {Ψd(z,z)),+Ψ(d(Sz, Sz))}

+ C.min {Ψd(z, Sz)), Ψ(d(Sz, z))}

So that Ψ(d (Sz,z)) $\leq$ (A+C). Ψ(d(Sz,z)),

which is a contradiction, since (A+C) > 1.

Hence Sz = z.i.e.z is a fixed point of S.

Thus z is a common fixed point of I, S and T if I weak ** commutes with S. Similarty we can prove that z is a common fixed point of I, S and T, if is weak ** commutes with T and S is rotative w.r. to I.

If we assume that S is weak ** commutes with I, then as above we can

show that, Iz = z = Sz and T^2z= z

If T is rotative w.r. to S, we have

Ψ (d(Tz,z)) = Ψ(d(Tz,S^2z)) $\leq$ Ψ(d(Sz,T^2z))= Ψ(d(z,z)) = 0

Hence Tz=z. Thus z is a common fixed point of I, S and T if S is weak ** commuting with I and T rotative w.r.t.S.

Proceeding in the same way, we can show that z is a common fixed point of I,S and T if T is weak ** commuting with I and S is rotative w.r. to T.

If z' is another common fixed point of S and I then we get

I^2z' = z' and S^2z' =z' if S^2z' $\neq$ I^2z ,

then Ψ(d(I^2z, S^2z')) = Ψ(d(z,S^2z'))

$= \Psi(d(T^2w, S^2z'))$

$\leq$ A. max. $\{ \Psi(I^2w, I^2z')), [^1/_2. \Psi(d(I^2w,I^2z')). \Psi(d(I^2z',T^2w))]^{1/2}\}$

$+ B. \{ \Psi(d(I^2w, T^2w)) + \Psi(d(I^2z', S^2z'))\}$

$+ C.min. \{ \Psi(d(I^2w, S^2z')), \Psi(d(I^2z', T^2w))\}$

i.e. $\Psi(d(z,z')) \leq$ A. max.$\{\Psi(d(z,z')), [^1/_2. \Psi(d(z',z)). \Psi(d(z',z))]^{1/2}\}$

$+ B.\{ \Psi(d(z,z)) + \Psi(d(z',z))\}$

$+ C. min. \{ \Psi(d(z,z')), \Psi(d(z',z))$.

That is $\Psi (d(z', z) \leq (A+C) . \Psi(d(z,z'))$.

This is a contradiction, since (A+C) <1. So that $S^2z' =I^2z$ i.e. z' =z,

Hence z is a unique common fixed point of S and I.

We can prove similarly that z is a unique common fixed point of I and T and also for S and T.

Assuming S = T on X, we have the following Corollary.

Corollary

Let S and I be mappings of a complete metric space (X,d) in to itself such that for x, y in X,

(V) $\Psi(d(S^2x,S^2y)) \leq$ A. max.$\{\Psi(d(I^2x,I^2y)), [^1/_2. \Psi(d(I^2x,I^2y). \Psi(d(I^2y,S^2x))]^{1/2}\}$

$+ B. \{\Psi(d(I^2x, S^2x)) + \Psi(d(I^2y, S^2y))\}$

$+ C. min.\{ \Psi(d(I^2x, S^2y)), \Psi(d(I^2y,S^2x))\}$,

where $(A+2B+C) < 1$, for $A,B,C \geq 0$.

If the range of I^2 contains the range of S^2, if I weak ** commutes with S and if S^2 or I^2 is continuous, then S and I have a unique common fixed point.

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