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Use of Chebyshev Polynomials in Digital Processing Of Signals

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ABSTRACT. The article presents the receipt and the grouping of spectral coefficients of signal processing and images by using the spectral method based on Wavelet-Haar basis matrix and the coefficients of orthogonal Chebyshev's polynomials.

KEYWORDS: signal processing, interpolation, orthogonal polynomials, spectral coefficients, basis matrix, Wavelet-Haar, Uolsh – Adamar, Chebyshev's polynomials, elementary functions.

1. INTRODUCTION

Signal and images filtration in spectral sphere, multiplexing, new mathematical method and algorithm creation of interpolation and decimation as well as the software development for their special processors and network computers are considered as a culmination problem in these days. In this case geodesy, cartography spheres, space research and the digital process of medical images can be taken as a exploration object. These questions in the same time are learned and handled in timing or spectral sphere. If the research goes in the timing sphere, the quality of created algorithm data will be decreased, other than that in spectral sphere such kind of idiosyncrasies not only will be explored but incognito information will also be found. In this article the receipt and the grouping of spectral coefficients of signal processing and images by using the spectral method based on Wavelet-Haar basis matrix and the coefficients of orthogonal Chebyshev's polynomials are presented [3], [4], [5].

II. THE PURPOSE OF THE TASK

Chebyshev orthogonal polynomials accidental central value is evaluated with the step $x_{k+1} - x_k = h \quad (k = 1,2,..., N-1)$ by adding variable $u = \dfrac{x - \bar{x}}{h}$, here is $\bar{x} = \dfrac{x_1 + x_N}{2}$ [1].

Polynomial appearance is expressed like following:

$$\varphi(x) = c_0 p_0(u) + c_1 p_1(u) + ... + c_n p_n(u) \qquad (1)$$

and

$$c_j = \frac{1}{H_j} \sum_{k=1}^{N} \varphi_k p_j(u_k) \quad (j = 0,1,2,\dots,n;\ n < N) \qquad (2)$$

by this formula coefficient is evaluated. Here $H_j = \sum_{k=1}^{N} p_j^2(u_k)$, $u_k = \dfrac{x_k - \bar{x}}{h}$ only receives integer values (for odd numbers $0, \pm1, \pm2, \dots, \pm\dfrac{N-1}{2}$, for even numbers $\pm\dfrac{1}{2}, \pm\dfrac{3}{2}, \dots, \pm\dfrac{N-1}{2}$).

Chebyshev orthogonal polynomials beginning and first coefficients:

$$p_0(u) = 1, \quad p_1(u) = u \qquad (3)$$

and others are found by following recurrent formulas:

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Vol. 6, Issue 2, February 2019

$$p_{j+1}(u)=up_j(u)-\frac{H_j}{H_{j-1}}p_{j-1}(u),\quad j=1,2,\dots \tag{4}$$

H_j and $\frac{H_j}{H_{j-1}}$ in this formula are calculated by following:

$$H_j=\frac{(j)^2(N+j)(N+j-1)\dots(N-j)}{4^j\left[(2j-1)!!\right]^2(2j+1)},$$

$$\frac{H_j}{H_{j-1}}=\frac{j^2\left(N^2-j^2\right)}{4(2j-1)(2j+1)}\qquad (j=1,2,3,\dots) \tag{5}$$

here is $(2j-1)!!=1\cdot 3\cdot 5\dots(2j-1)$. For example,

$$H_0=N\,,\ H_1=\frac{N\left(N^2-1\right)}{12},\ H_2=\frac{N\left(N^2-1\right)\left(N^2-4\right)}{180},\dots$$

$$\frac{H_1}{H_0}=\frac{N^2-1}{12},\ \frac{H_2}{H_1}=\frac{N^2-4}{15},\ \frac{H_3}{H_2}=\frac{\left(N^2-9\right)\cdot 9}{140},\dots$$

8 terms of orthogonal polynomials are formulated by following appearance:

$$p_0(u)=1,\ p_1(u)=u\,,\ p_2(u)=u^2-5\frac{1}{4},$$

$$p_3(u)=u^3-9\frac{1}{4}u\,,\ p_4(u)=u^4-\frac{179}{14}u^2+\frac{297}{16}$$

for $k=3$ and $N=8$

$$\varphi(u)=\left(c_0-\frac{21}{4}c_2\right)+\left(c_1-\frac{37}{4}c_3\right)u+c_2u^2+c_3u^3 \tag{6}$$

In order to renew and analyse the given signal, this should be taken in the following algebraic polynomial form:

$$\varphi(t)=\sum_{k=0}^{n}A_kt^k \tag{7}$$

Input signal spectre $W_s(t)$ is formulated by the following formula with the help of A_k coefficient in the Uolsh – Adamar basis:

$$a_s=\sum_{k=0}^{n}A_kb_s^k\qquad (s=0,1,2,\dots) \tag{8}$$

here b_s^k - t^k are spectral coefficients of grade polynomial [2].

By using spectral method A_k coefficient can be calculated efficiently. For this with the help of (1) formula and by using basis $W_s(t)$ and Parseval's theorem following form will be taken:

$$A_k=\frac{N}{H_k}\sum_{s=0}^{N-1}a_sc_s^k \tag{9}$$

here c_s^k - $p_k(t)$ is Chebyshev's polynomial spread spectral coefficient in $W_s(t)$ basis.

It is obvious by (6) and (7) formulas, if there were $k=3$ and $N=8$

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$$\begin{aligned} A_0 &= c_0 - \frac{21}{4}c_2 \\ A_1 &= c_1 - \frac{37}{4}c_3 \\ A_2 &= c_2 \\ A_3 &= c_3 \end{aligned} \qquad (10)$$

would be originated.

Chebyshev's polynomial basis matrix in $[-1;1]$ interval on basis of (4) form will be like following [1], [3]:

$$\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -3{,}5 & -2{,}5 & -1{,}5 & -0{,}5 & 0{,}5 & 1{,}5 & 2{,}5 & 3{,}5 \\ 7 & 1 & -3 & -5 & -5 & -3 & 1 & 7 \\ -10{,}5 & 7{,}5 & 10{,}5 & 4{,}5 & -4{,}5 & -10{,}5 & -7{,}5 & 10{,}5 \end{pmatrix}$$

c_s^k coefficients $p_k(t)$ matrix when spreading by $W_s(t)$ basis matrix is created like that:

$$\begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -2 & -1 & 0 & -0{,}5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 2 & 1 & 0 \\ 0 & 3 & -4{,}5 & 0 & -3 & 0 & 0 & -6 \end{pmatrix}$$

By using (9) and (5) formulas

$$\begin{aligned} A_3 &= \frac{1}{99}(2a_1 - 3a_2 - 2a_5 - 4a_7) \\ A_2 &= \frac{1}{21}(4a_3 + 2a_5 + a_6) \\ A_1 &= -\frac{2}{21}(4a_1 + 2a_2 + a_4) - \frac{37}{4}A_3 \\ A_0 &= a_0 - \frac{21}{4}A_2 \end{aligned} \qquad (11)$$

is realized .

By these forms output signal's analytic is found:

$$F(x) = A_0 + A_1 u + A_2 u^2 + A_3 u^3,$$

here $u = \dfrac{x - \bar{x}}{h}$, $\bar{x} = \dfrac{x_1 + x_N}{2}$.

Mixed Chebyshev's polynomial in $[0;1]$ interval is calculated by following[5]:

$$T_n^*(x) = T_n(2x-1) = \cos n\vartheta \qquad (\cos n\vartheta = 2x - 1;\ n = 0,1,2,\ldots)$$

or

$$T_0^*(x) = 1, \quad T_1^*(x) = 2x - 1, \quad T_{n+1}^*(x) = 2(2x-1)T_n^*(x) - T_{n-1}^*(x) \qquad (n = 1,2,\ldots).$$

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III. FUNCTION MODULE

Even when using this and (9) expressions for k=3 and N=8 the accuracy of (11) formulas will be originated.

With the help of Wavelet – Haar change [4] and Chebyshev's polynomial input $\varphi(x)$ signal when being transferred to polynomial form when aforementioned method is used

$$\begin{aligned} A_3 &= \frac{1}{99}\left(4v_1 - 3v_2 - 3v_3 - 3v_4 + v_5 + v_6 - 3v_7\right) \\ A_2 &= \frac{1}{21}\left(2v_2 - 2v_3 + \frac{3}{4}v_4 + \frac{1}{4}v_5 - \frac{1}{4}v_6 - \frac{3}{4}v_7\right) \\ A_1 &= -\frac{1}{21}\left(8v_1 + 2v_2 + 2v_3 + \frac{1}{2}v_4 + \frac{1}{2}v_5 + \frac{1}{2}v_6 + \frac{1}{2}v_7\right) - \frac{37}{4}A_3 \\ A_0 &= v_0 - \frac{21}{4}A_2 \end{aligned} \qquad (12)$$

will be originated .

By the same method, with the help of Wavelet – Haar change [4] and Chebyshev's polynomial input $\varphi(x)$ signal when being transferred to polynomial form

$$\begin{aligned} A_3 &= \frac{1}{99}\left(4h_1 - 3\sqrt{2}h_2 - 3\sqrt{2}h_3 - 6h_4 + 2h_5 + 2h_6 - 6h_7\right) \\ A_2 &= \frac{1}{21}\left(2\sqrt{2}h_2 - 2\sqrt{2}h_3 + \frac{3}{2}h_4 + \frac{1}{2}h_5 - \frac{1}{2}h_6 - \frac{3}{2}h_7\right) \\ A_1 &= -\frac{1}{21}\left(8h_1 + 2\sqrt{2}h_2 + 2\sqrt{2}h_3 + h_4 + h_5 + h_6 + h_7\right) - \frac{37}{4}A_3 \\ A_0 &= h_0 - \frac{21}{4}A_2 \end{aligned} \qquad (13)$$

will be created.

IV. CONCLUSION

Taken polynomial coefficients with the help of (11), (12) and (13) expressions are respectively equal which was proved in practice. By this it can be concluded that, by this method no matter what kind of basis matrix and change are used, any way traceable polynomial coefficients are respectively equal.

As well as, in (11) and (12) forms spectral coefficients are used by 8. In order to find Wavelet-Haar changeable spectral coefficients 24 summation and 14 shift (multiple operation replacement in numerical registers) operations are completed. In order to find Uolsh-Adamar changeable spectral coefficients 89 summation and 24 shift operations are completed. This in the same time in the aforementioned method, Wavelet-Haar change usage gives opportunity to win by time and decreases the call to the memory. The usage of these methods elementary functions (cos$x$, sin$x$, ln$x$, log$x$, x^n, $\sqrt{x}$, e^x) for signal processors formulating in the form of polynomial differ from other methods by lack of operations (Taylor's row, Makloren's row, limited differential method, the most small quadratic method and many more).

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