ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 7, Issue 10 , October 2020

AHM: A Measure of the Value of Parameter μ of the Model $X = \mu + \varepsilon$

Dhritikesh Chakrabarty

Associate Professor, Department of Statistics, Handique Girls' College, Guwahati – 781001, Assam, India

ABSTRACT: In continuation to the study on formulation of arithmetic–geometric mean (abbreviated as *AGM*) by Gauss, which has recently been found to be a technique of evaluating the value of parameter from observed data containing the parameter itself and random error, an attempt has here been made on formulating of one measure of average termed as arithmetic–harmonic mean (abbreviated as *AHM*) with an attempt to derive that this formulation can be a technique of determining the value of parameter from observed data containing itself and random error. This paper describes the formulation of *AHM* and the derivation of the technique along with numerical application.
.

I. INTRODUCTION

There had been lot of researches on the construction of tables of random numbers by reputed researchers like *Tippett* Several research have already been done on developing definitions of average [1 , 2], a basic concept used in developing most of the measures used in analysis of data. Pythagoras [3], the pioneer of researchers in this area, constructed three definitions / formulations of average namely Arithmetic Mean, Geometric Mean & Harmonic Mean which are called Pythagorean means [4 , 5 , 14 , 18]. A lot of definitions / formulations have already been developed among which some are arithmetic mean. geometric mean, harmonic mean, quadratic mean, cubic mean, square root mean, cube root mean, general p mean and many others [6 − 19]. Kolmogorov [20] formulated one generalized definition of average namely Generalized f - Mean. [7 , 8]. It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized f - Mean [9 , 10]. In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean [11]. In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean [12 , 13] and developed one general method of defining average [15 – 17] as well as the different formulations of average from the first principles [19].

In many real situations, observed numerical data

$$x_1 , x_2 , \ldots\ldots , x_n$$

are found to be composed of a single parameter μ and corresponding chance / random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

i.e. the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots , N$$

[21 – 29].

The existing methods of estimation of the parameter μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [31 – 52] cannot provide appropriate value of the parameter μ [21 – 23]. In some recent studies, some methods have been developed for determining the value of parameter from observed data containing the parameter itself and random error [21 – 30 , 53 – 60]. In continuation to the study on formulation of average starting from Pythagorean means, Gauss developed one formulation of average from the definitions of arithmetic mean and geometric mean. This definition later on was termed as arithmetic–geometric mean (abbreviated as *AGM*) [61 – 62]. Recently, this formulation of average (namely *AGM*) has been applied in evaluating the value of parameter from observed data containing the parameter itself and random error [63 – 64].

In continuation to the study on formulation of arithmetic–geometric mean (abbreviated as *AGM*) by Gauss, which has recently been found to be a technique of evaluating the value of parameter from observed data containing the parameter itself and random error, an attempt has here been made on formulating of one measure of average termed as arithmetic–harmonic mean (abbreviated as *AHM*) with an attempt to derive that this formulation can be a technique of determining the value of parameter from observed data containing itself and random error. This paper describes the formulation of *AHM* and the derivation of the technique along with numerical application.

II. ARITHMETIC-HARMONIC MEAN (AHM)

Let a_0 & h_0 be respectively the *AM* (Arithmetic Mean) & *HM* (Harmonic Mean) of the N numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots , x_N$$

From the inequality of Pythagorean means [4 , 5] namely

$$AM > GM > HM$$

(where *GM* means Geometric Mean),
it follows that

$$a_0 > h_0$$

provided $x_1 , x_2 , \ldots\ldots , x_N$ are not all equal.
Let $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{\boldsymbol{h}'_n = \boldsymbol{h}'_n(a_0 , h_0)\}$ be two sequences defined by

$$a'_{n+1} = ½(a'_n + \boldsymbol{h}'_n)$$

$$\& \quad \boldsymbol{h}'_{n+1} = ½(a'^{-1}_n + \boldsymbol{h}'^{-1}_n)\}^{-1}$$

respectively.
It is obvious that

$$a'_0 = a'_0(a_0 , h_0) = a_0 \quad \& \quad \boldsymbol{h}'_0 = h'_0(a_0 , h_0) = h_0$$

By the inequality of Pythagorean means [4 , 5] ,

$$\boldsymbol{h}'_n < a'_n$$

and thus $\quad a'_{n+1} = ½ (a'_n + \boldsymbol{h}'_n)$

$$\Rightarrow \quad a'_{n+1} < ½(a'_n + a'_n)$$

$$\Rightarrow \quad a'_{n+1} < a'_n$$

This means that the sequence $\{a'_n = a'_n(a_0 , h_0)\}$ is non-increasing.
Moreover, the sequence $\{a'_n = a'_n(a_0 , h_0)\}$ is bounded below by the smallest of

$$x_1 , x_2 , \ldots\ldots , x_N .$$

Therefore, by the monotone convergence theorem [65 , 66], the sequence is convergent.
Therefore, there exists a finite number M_{AH} such that

$$a'_n \text{ converges to } M_{AH} \text{ as } n \text{ approaches infinity.}$$

Again, $\boldsymbol{h}'_n$ can be expressed as

$$\boldsymbol{h}'_n = 2a'_{n+1} - a'_n$$

This implies that the limiting value of $\boldsymbol{h}'_n$ as n approaches infinity is M_{AH} .
Therefore,

$$\boldsymbol{h}'_n \text{ converges to } M_{AH} \text{ as } n \text{ approaches infinity.}$$

Thus, the two sequences $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{\boldsymbol{h}'_n = \boldsymbol{h}'_n(a_0 , h_0)\}$ converge to the same point M_{AH} as n approaches infinity.
This common converging point M_{AH} can be termed / named / regarded as the Arithmetic-Harmonic Mean (abbreviated as *AHM*) of the N numbers (or values or observations)

$$x_1 , x_2 , \ldots\ldots , x_N$$

III. AHM AS A TECHNIQUE OF EVALUATION OF μ

If the observations

$$x_1 , x_2 , \ldots\ldots , x_N$$

are composed of some parameter μ and random errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots , N)$$

where

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 7, Issue 10 , October 2020

are the random errors, which assume positive and negative values in random order, associated to

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

respectively.
In this case,

$$A(x_1 , x_2 , \ldots\ldots\ldots , x_N) \to \mu \text{ as } N \to \infty$$

where $A(x_1 , x_2 , \ldots\ldots\ldots , x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i$

Again since the observations

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

consist of μ and random errors,
therefore, the reciprocals

$$x_1^{-1} , x_2^{-1} , \ldots\ldots\ldots\ldots , x_N^{-1}$$

are composed of μ^{-1} and random errors different from the respective random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots\ldots , \varepsilon_N$$

provided $x_1 , x_2 , \ldots\ldots\ldots , x_N$ are all different from zero.
In this case thus

$$x_i^{-1} = \mu^{-1} + \varepsilon_i^{/} \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , N)$$

where

$$\varepsilon_1^{/} , \varepsilon_2^{/} , \ldots\ldots\ldots\ldots\ldots , \varepsilon_N^{/}$$

are the random errors, which assume positive and negative values in random order, associated to are the random errors associated to

$$x_1^{-1} , x_2^{-1} , \ldots\ldots\ldots\ldots , x_N^{-1}$$

respectively..
In this case,

$$H(x_1 , x_2 , \ldots\ldots\ldots , x_N) \to \mu \text{ as } N \to \infty$$

where $H(x_1 , x_2 , \ldots\ldots\ldots , x_N) = (\frac{1}{N} \sum_{i=1}^{N} x_i^{-1})^{-1}$

This implies that the common converging value of

$$A(x_1 , x_2 , \ldots\ldots\ldots , x_N) \;\&\; H(x_1 , x_2 , \ldots\ldots\ldots , x_N)$$

is the value of μ.
It is to be noted that the converging value may not be possible to be obtained for a finite set of observed values namely

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

In order to obtain the value of μ, in this case, let us write

$$A(x_1 , x_2 , \ldots\ldots\ldots , x_N) = A_0$$
$$\& \; H(x_1 , x_2 , \ldots\ldots\ldots , x_N) = H_0$$

and then define the two interdependent sequences $\{A_n\}$ and $\{H_n\}$ as

$$A_{n+1} = ½\,(A_n + H_n)$$
$$\& \quad H_{n+1} = \{½\,(A_n^{-1} + H_n^{-1})\}^{-1}$$

Then, both of A_n & H_n converges to some real number C as n approaches infinity.
Now, it is required to verify whether this C is equal to μ.
From the model it is obtained that

$$A_0 = \mu + \delta_0 \quad \& \quad H_0 = \mu + e_0$$

The inequality of Pythagorean means namely

$$\text{AM} > \text{HM}$$

implies that $A_0 > H_0$ i.e. $\delta_0 > e_0$
Thus $A_1 = \mu + \delta_1$ where $\delta_1 = ½\,(\delta_0 + e_0) < \delta_0$
In general, corresponding to A_{n+1}, it holds that

$$\delta_{n+1} = ½\,(\delta_n + e_n) < \delta_n$$

This implies, δ_n converges to 0 i.e. A_n converges to μ.
By the existence of AHM, H_n also converges to μ.
Thus, the *AHM* of

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

is the value of μ. .

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 7, Issue 10 , October 2020

IV. NUMERICAL EXAMPLE: APPLICATION TO NUMERICAL DATA

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate the central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati

A. Annual Maximum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013 [63 , 64], the *AM* & the *HM* have been found to be

37.2093023255814 & 37.175398903562627634836294491501

respectively.

Here the observed values can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and random errors.

Evaluation of Value of μ (the central tendency of annual maximum)

Let us write

$$A_0 = 37.2093023255814 \quad \& \quad H_0 = 37.175398903562627634836294491501$$

In this case the iterations give the values which are given in the following table (**Table – 1**):

Table – 1

n	A_n	H_n
0	37.2093023255814	37.175398903562627634836294491501
1	37.192350614572013817418147245751	37.183872827043641055199463981829
2	37.188111720807827436308805613779	37.188112376367409623363572976814
3	37.188111479222284199322581455736	37.188111479222282629907723330485
4	37.188111479222283414615152393111	37.188111479222283022261437861789
5	37.188111479222283218438295127745	37.188111479222283218438295127448
6	37.188111479222283218438295127449	37.188111479222283218438295127449

The digits in A_n and H_n, which are agreed, have been underlined in the above table.

The *AHM* of

37.2093023255814 & 37.175398903562627634836294491501

is the common limit of these two sequences which is 37.188111479222283218438295127449

Thus the value of μ, the central tendency of annual maximum of surface air temperature at Guwahati, obtained by *AHM*, is 37.188111479222283218438295127449 Degree Celsius.

B. Annual Minima of Surface Air Temperature at Guwahati

From the observed data on annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013 [63 , 64], the *AM* & the *HM* have been found to be

7.3634146341463414634146341463415 & 7.1543933802823525209849744707569

respectively.

In this case also, the observed values can be assumed to be composed of a parameter μ (representing the central tendency of annual maximum) and random errors.

Determination of Value of μ (the central tendency of annual minimum)

In this case the iterations give the values which are given in the following table (**Table – 2**):

ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 7, Issue 10 , October 2020

Table – 2

n	A_n	H_n
0	7.36341463414634146341463415	7.1543933802823525209849744707569
1	7.2589040072143469921998043085492	7.2573993074510131470508335954916
2	7.2581516573326800696253189520204	7.2581515793472133602889381001075
3	7.258151618339946714957128526064	7.2581516183399465054777273757201
4	7.2581516183399466102174279508921	7.2581516183399466102174279508919
5	7.258151618339946610217427950892	7.258151618339946610217427950892

The digits in A_n and H_n, which are agreed, have been underlined in the above table.

The AHM of

7.36341463414634146341463415 & 7.1543933802823525209849744707569

is the common limit of these two sequences which is 7.258151618339946610217427950892

Thus the value of μ, the central tendency of annual minimum of surface air temperature at Guwahati, obtained by *AHM*, is 7.258151618339946610217427950892 Degree Celsius.

V. CONCLUSION

In the methods developed so far, for determining the value of parameter from observed data containing the parameter itself and random error, a finite set of observed data may not be sufficient for obtaining the value of the parameter. However, the application of AHM can yield the value of the parameter even if the set of observed data is small.
Moreover, the application of AHM in determining the value of parameter in this situation involves lesser computational tasks than those involved in the methods developed so far for the same purpose.
It seems that there is scope of developing more formulation(s) of average based on the other combinations of the three Pythagorean means namely arithmetic mean, geometric mean and harmonic mean.

REFERENCES

[1]. Bakker Arthur, "The early history of average values and implications for education", *Journal of Statistics Education*, 2003, 11(1), 17 – 26.
[2]. Miguel de Carvalho, "Mean, what do you Mean?", *The American Statistician*, 2016, 70, 764 – 776.
[3]. Christoph Riedweg, "*Pythagoras: his life, teaching, and influence* (translated by Steven Rendall in collaboration with Christoph Riedweg and Andreas Schatzmann, Ithaca)", ISBN 0-8014-4240-0, 2005, Cornell University Press.
[4[. David W. Cantrell, *"*Pythagorean Means*", Math World*.
[5]. Dhritikesh Chakrabarty, "Pythagorean Mean: Concept behind the Averages and Lot of Measures of Characteristics of Data", NaSAEAST- 2016, *Abstract ID: CMAST_NaSAEAST (Inv)-1601)*. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
[6]. Dhritikesh Chakrabarty, "Objectives and Philosophy behind the Construction of Different Types of Measures of Average", NaSAEAST- 2017, *Abstract ID: CMAST_NaSAEAST (Inv)-1701)*, Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
[7]. Andrey Kolmogorov , "On the Notion of Mean", in "*Mathematics and Mechanics*" (Kluwer 1991), 1930, 144 – 146.
[8]. Andrey Kolmogorov, *"Grundbegriffe der Wahrscheinlichkeitsrechnung (in German)*, 1933, Berlin: Julius Springer.
[9]. Dhritikesh Chakrabarty, "Derivation of Some Formulations of Average from One Technique of Construction of Mean", *American Journal of Mathematical and Computational Sciences*, 2018, 3(3), 62 – 68. Available at http://www.aascit.org/journal/ajmcs.
[10]. Dhritikesh Chakrabarty, "One Generalized Definition of Average: Derivation of Formulations of Various Means", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN: 2278 – 179 X), 2018, 7(3), 212 – 225. Available at www.jecet.org.
[11]. Dhritikesh Chakrabarty, "f_H -Mean: One Generalized Definition of Average", *Journal of Environmental Science Computer Science and Engineering & Technology*, Section C, (E-ISSN: 2278 – 179 X), 2018, 7(4), 301 – 314. Available in www.jecet.org.
[12]. Dhritikesh Chakrabarty, "Generalized f_G - Mean: Derivation of Various Formulations of Average", *American Journal of Computation, Communication and Control*, 2018, 5(3), 101 – 108. Available at http://www.aascit.org/journal/ajmcs .
[13]. Dhritikesh Chakrabarty, "One Definition of Generalized f_G - Mean: Derivation of Various Formulations of Average", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E- ISSN : 2278 – 179 X), 2019, 8(2), 051 – 066. Available at www.jecet.org.
[14]. Dhritikesh Chakrabarty, "Pythagorean Mean: Concept behind the Averages and Lot of Measures of Characteristics of Data", NaSAEAST- 2016, *Abstract ID: CMAST_NaSAEAST (Inv)-1601)*, 2016. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
[15]. Dhritikesh Chakrabarty, "General Technique of Defining Average", NaSAEAST- 2018, *Abstract ID CMAST_NaSAEAST -1801 (I)*, Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .
[16]. Dhritikesh Chakrabarty, "One General Method of Defining Average: Derivation of Definitions/Formulations of Various Means", *Journal of*

Environmental Science, Computer Science and Engineering & Technology, Section C, (E-ISSN : 2278 – 179 X), 2019, 8(4), 327 – 338. Available at www.jecet.org .

[17]. Dhritikesh Chakrabarty, "A General Method of Defining Average of Function of a Set of Values", *Aryabhatta Journal of Mathematics & Informatics* {ISSN (Print) : 0975-7139, ISSN (Online) : 2394-9309}, 2019, 11(2), 269 – 284. Available at www.abjni.com .

[18]. Dhritikesh Chakrabarty, "Pythagorean Geometric Mean: Measure of Relative Change in a Group of Variables", NaSAEAST- 2019, *Abstract ID: CMAST_NaSAEAST -1902 (I)*, 2019. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[19]. Dhritikesh Chakrabarty, "Definition / Formulation of Average from First Principle", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 2020, 9(2), 151 – 163. Available at www.jecet.org .

[20]. A. P. Youschkevitch , *"*A. N. Kolmogorov: Historian and philosopher of mathematics on the occasion of his 80th birfhday*", Historia Mathematica*, 1983, 10(4), 383 – 395.

[21]. Dhritikesh Chakrabarty, "Determination of Parameter from Observations Composed of Itself and Errors", *International Journal of Engineering Science and Innovative Technology*, (ISSN: 2139 – 5967), 2014, 3(2), 304 – 311. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[22]. Dhritikesh Chakrabarty, "Analysis of Errors Associated to Observations of Measurement Type", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2014, 1(1), 15 – 28. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[23]. Dhritikesh Chakrabarty, "Observation Composed of a Parameter and Random Error: An Analytical Method of Determining the Parameter", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2014, 1(2), 20 – 38, 2014. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[24]. Dhritikesh Chakrabarty, "Observation Consisting of Parameter and Error: Determination of Parameter", *Proceedings of the World Congress on Engineering* 2015, (WCE 2015, July 1 - 3, 2015, London, U.K.), ISBN: 978-988-14047-0-1, ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online), 2015, Vol. II, 680 – 684. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[25]. Dhritikesh Chakrabarty: "Observation Consisting of Parameter and Error: Determination of Parameter," Lecture Notes in Engineering and Computer Science (ISBN: 978-988-14047-0-1), London, 2015, 680 – 684. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[26]. Dhritikesh Chakrabarty, "Central Tendency of Annual Extremum of Surface Air Temperature at Guwahati", *J Chem. Bio. Phy. Sci.* (E- ISSN : 2249 – 1929), Sec. C, 2015, 5(3), 2863 – 2877. Available at: www.jcbsc.org.

[27]. Dhritikesh Chakrabarty, :Central Tendency of Annual Extremum of Surface Air Temperature at Guwahati Based on Midrange and Median", *J. Chem. Bio. Phy. Sci.* (E- ISSN : 2249 –1929), Sec. D, 2015, 5(3), 3193 – 3204. Available at: www.jcbsc.org.

[28]. Dhritikesh Chakrabarty, "Observation Composed of a Parameter and Random Error: Determining the Parameter as Stable Range", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2015, 2(1), 35 – 47. Available at *http://eses.net.in/ESES Journal*.

[29]. Dhritikesh Chakrabarty, "A Method of Finding Appropriate value of Parameter from Observation Containing Itself and Random Error", *Indian Journal of Scientific Research and Technology*, (E-ISSN: 2321-9262), 2015, 3(4), 14 – 21. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[30]. Dhritikesh Chakrabarty, "Theoretical Model Modified For Observed Data: Error Estimation Associated to Parameter", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2015, 2(2), 29 – 45. Available at *http://eses.net.in/ESES Journal*.

[31]. Anders Hald, "On the History of Maximum Likelihood in Relation to Inverse Probability and Least Squares", *Statistical Science*, 1999, 14, 214 – 222.

[32]. Barnard G. A., "Statistical Inference", *Journal of the Royal Statistical Society, Series B*, 1949, 11, 115 – 149.

[33]. Birnbaum Allan, "On the Foundations of Statistical Inference", *Journal of the American Statistical Association*, 1962, 57, 269 – 306.

[34]. Ivory, "On the Method of Least Squares", *Phil. Mag*., 1825, LXV, 3 – 10.

[35]. Kendall M. G. and Stuart A, "*Advanced Theory of Statistics*", *Vol. 1 & 2, 4th Edition, New York, Hafner Press*, 1977.

[36]. Lehmann Erich L. & Casella George, *Theory of Point Estimation, 2nd ed. Springer.* ISBN 0 – 387 – 98502 – 6, 1998.

[37]. Lucien Le Cam, "Maximum likelihood — An introduction", *ISI Review*, 1990, 8 (2), 153 – 171.

[38]. Walker Helen M. & Lev J., "*Statistical Inference*", Oxford & IBH Publishing Company, 1965.

[39]. Dhritikesh Chakrabarty & Atwar Rahman, "Exponential Curve : Estimation Using the Just Preceding Observation in Fitted Curve", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2007, 3(2), 381 – 386. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[40]. Dhritikesh Chakrabarty & Atwar Rahman, "Gompartz Curve : Estimation Using the Just Preceding Observation in Fitted Curve", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2008, 4(2), 421 – 424. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[41].Rahman Atwar & Dhritikesh Chakrabarty, "Linear Curve : A Simpler Method of Obtaining Least squares Estimates of Parameters", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2009, 5(2), 415 – 424. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[42]. Dhritikesh Chakrabarty, "Finite Difference Calculus: Method of Determining Least Squares Estimates", *AryaBhatta J. Math. &Info.* (ISSN : 0975 – 7139), 2011, 3(2), 363 – 373. Available at www.abjni.com. Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[43]. Atwar Rahman & Dhritikesh Chakrabarty, "General Linear Curve : A Simpler Method of Obtaining Least squares Estimates of Parameters", *Int. J. Agricult. Stat. Sci.*, (ISSN : 0973 - 1903), 2011, 7(2), 429 – 440. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats

[44]. Dhritikesh Chakrabarty, "Curve Fitting: Step-Wise Least Squares Method", *AryaBhatta J. Math. &Info.*, 2014, 6(1), (ISSN : 0975 – 7139), 15 – 24. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[45]. Atwar Rahman & Dhritikesh Chakrabarty, "Elimination of Parameters and Principle of Least Squares: Fitting of Linear Curve to Average Minimum Temperature Data in the Context of Assam", *International Journal of Engineering Sciences & Research Technology*, 4(2), (ISSN : 2277 - 9655), 2015, 255 – 259. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[46]. Atwar Rahman & Dhritikesh Chakrabarty, "Elimination of Parameters and Principle of Least Squares: Fitting of Linear Curve to Average Maximum Temperature Data in the Context of Assam", *AryaBhatta J. Math. & Info.* ISSN (Print): 0975 – 7139, ISSN (Online): 2394 – 9309), 2015, 7(1), 23 – 28. Available at www.abjni.com . Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[47[. Atwar Rahman & Dhritikesh Chakrabarty, "Basian-Markovian Principle in Fitting of Linear Curve", The *International Journal Of Engineering And Science*, {ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805}. Available at www.theijes.com), 2015, 4(6), 31 – 43. Also Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[48]. Atwar Rahman & Dhritikesh Chakrabarty, "Basian-Markovian Principle in Fitting of Quadratic Curve", *International Research Journal of Natural and Applied Sciences* (ISSN: 2349 - 4077), www.aarf.asia, 2015, 2(6), 86 – 210. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[49]. Atwar Rahman & Dhritikesh Chakrabarty, "Method of Least Squares in Reverse Order: Fitting of Linear Curve to Average Maximum Temperature Data at Guwahati and Tezpur", *International Journal in Physical & Applied Sciences* (ISSN: 2394 - 5710), www.ijmr.net.in, 2015, 2(9), 24 – 38. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[50]. Atwar Rahman & Dhritikesh Chakrabarty, "Method of Least Squares in Reverse Order: Fitting of Linear Curve to Average Minimum Temperature Data at Guwahati and Tezpur", ,*AryaBhatta J. Math. & Info.* {ISSN (Print): 0975 – 7139, ISSN (Online): 2394 – 9309}, 2015, 7(2), 305 – 312. Available at www.abjni.com . Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[5]1. Dhritikesh Chakrabarty, "Elimination-Minimization Principle: Fitting of Polynomial Curve to Numerical Data", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 2016, 3(5), 2067 – 2078. Available at www.ijarset.com. Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[52]. Dhritikesh Chakrabarty, "Elimination-Minimization Principle: Fitting of Exponential Curve to Numerical Data", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 2016, 3(6), 2256 – 2264. *Available at www.ijarset.com*. Also available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[53]. Dhritikesh Chakrabarty, "Impact of Error Contained in Observed Data on Theoretical Model: Study of Some Important Situations", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 2016, 3(1), 1255 – 1265. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[54]. Dhritikesh Chakrabarty, "Theoretical Model and Model Satisfied by Observed Data: One Pair of Related Variables", *International Journal of Advanced Research in Science, Engineering and Technology*, (ISSN : 2350 – 0328), 2016, 3(2), 1527 – 1534. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats .

[55]. Dhritikesh Chakrabarty, "Variable(s) Connected by Theoretical Model and Model for Respective Observed Data", FSDM2017, *Abstract ID: FSDM2220*, 2017. Available at https://www.researchgate.net/profile/Dhritikesh_Chakrabarty/stats.

[56]. Dhritikesh Chakrabarty, "Numerical Data Containing One Parameter and Random Error: Evaluation of the Parameter by Convergence of Statistic", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2017, 4(2), 59 – 73. Available at *http://eses.net.in/ESES Journal*.

[57]. Dhritikesh Chakrabarty, "Observed Data Containing One Parameter and Random Error: Evaluation of the Parameter Applying Pythagorean Mean", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2018, 5(1), 32 – 45. Available at *http://eses.net.in/ESES Journal*.

58. Dhritikesh Chakrabarty, "Significance of Change of Rainfall: Confidence Interval of Annual Total Rainfall", *Journal of Chemical, Biological and Physical Sciences* (E- ISSN : 2249 –1929), Sec. C, 2019, 9(3), 151 – 166. Available at: www.jcbsc.org.

[59]. Dhritikesh Chakrabarty, "Observed Data Containing One Parameter and Random Error: Probabilistic Evaluation of Parameter by Pythagorean Mean", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2019, 6(1), 24 – 40. Available at *http://eses.net.in/ESES Journal*.

[60]. Dhritikesh Chakrabarty, "Significance of Change in Surface Air Temperature in the Context of India", *Journal of Chemical, Biological and Physical Sciences* (E- ISSN : 2249 – 1929), Sec. C, 2019, 9(4), 251 – 261. Available at: www.jcbsc.org .

[61]. David A. Cox , "*The Arithmetic-Geometric Mean of Gauss*", *In J.L. Berggren; Jonathan M.Borwein; Peter Borwein (eds.). Pi: A Source Book. Springer. p. 481. ISBN 978-0-387-20571-7* , 2004, (first published in *L'Enseignement Mathématique*, t. 30 (1984), p. 275 – 330).

[62]. Hazewinkel, Michiel, ed. , "*Arithmetic–geometric mean process*, *Encyclopedia of Mathematics*", SpringerScience+Business Media B.V. / Kluwer Academic Publishers, *ISBN 978-1-55608-010-4*, 2001.

[63]. Dhritikesh Chakrabarty, "Arithmetic-Geometric Mean: Evaluation of Parameter from Observed Data Containing Itself and Random Error", *International Journal of Electronics and Applied Research* (ISSN : 2395 – 0064), 2019, 6(2), 98 – 111. Available at *http://eses.net.in/ESES Journal*.

[64]. Dhritikesh Chakrabarty, "AGM: A Technique of Determining the Value of Parameter from Observed Data Containing Itself and Random Error", *Journal of Environmental Science, Computer Science and Engineering & Technology*, Section C, (E-ISSN : 2278 – 179 X), 9(3), 2020, 473 – 486. Aavailable at www.jecet.org .

[65]. Weir, Alan J., "*The Convergence Theorems, Lebesgue Integration and Measure*", Cambridge: Cambridge University Press, 1973, 93 – 118. ISBN 0-521-08728-7.

[66].Yeh, J., "*Real Analysis: Theory of Measure and Integration",* Hackensack, NJ: World Scientific. ISBN 981-256- 653-8, *2006.*

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 7, Issue 10 , October 2020

AUTHOR'S BIOGRAPHY

Dr. Dhritikesh Chakrabarty, the author of this paper, passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Senior Diploma (in Guitar) from Prayag Sangeet Samiti in 2019 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

(Dr. Dhritikesh Chakrabarty in National Dong Hwa University, Hualien, Taiwan, presenting invited paper in The 3rd International Conference on Fuzzy Systems and Data Mining (FSDM 2017), November 24th – 27th, 2017)

Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.
Dr. Dhritikesh Chakrabarty is also an awardee of the Post Doctoral Research Award by the University Grants Commission for the period 2002─05.
He attended five of orientation/refresher course held in Gauhati University, Indian Statistical Institute, University of Calicut and Cochin University of Science & Technology sponsored/organized by University Grants Commission/Indian Academy of Science. He also attended/participated eleven workshops/training programmes of different fields at various institutes.

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
Vol. 7, Issue 10 , October 2020

Dr. Dhritikesh Chakrabarty joined the Department of Statistics of Handique Girls' College, Gauhati University, as a Lecturer on December 09, 1987 and has been serving the institution continuously since then. Currently he is in the position of Associate Professor (& Ex Head) of the same Department of the same College. He had also been serving the National Institute of Pharmaceutical Education & Research (NIPER), Guwahati, as a Guest Faculty continuously from May, 2010 to December,2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years. Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 228 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002─05) and one minor research project (2010─11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Referee of the Journal of Assam Science Society (JASS) and a Member of the Editorial Boards of the two Journals namely (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET) and (2) Journal of Mathematics and System Science. Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.