ISSN: 2350-0328

International Journal of Advanced Research in Science, Engineering and Technology

Vol. 8, Issue 12 , December 2021

Designing a Following System with Wave Channels Taking into Account the Measurement Noise and Disturbances on the System

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ABSTRACT: It is shown that using the results of measurements "clogged" with noise of linear combinations of object state variables, their estimates can be obtained by applying a filter consisting of a "model" of the original system and a feedback signal proportional to the difference between the actual measurement and its estimate. In this case, the optimal control law is found as a result of minimization of the ensemble-averaged quadratic functional of the system quality. A system is synthesized that has the property that, in the absence of other disturbances and noise of observations, the constant disturbance is always compensated in such a way that the control or tracking error in the steady state is zero, which is achieved due to the integrating action of the proportional-integral-differential controller. A method is proposed for calculating the control in a tracking system with wave channels, proceeding from the assumption that the full state vector is known (reconstructed based on the well-known separation theorem), and also proceeding from the replacement of the actual state of the system with the reconstructed one.

KEYWORDS: control actions; tracking system; wave channels; noise and signal environment.

I. INTRODUCTION

As you know, the optimal control of a dynamic system is necessary to know its state [1]. However, in practice, often individual variables of the optimal control state either cannot be measured directly, or are measured with a large error. Typically, the measurements that can be made are functions of state variables that contain random errors. As a rule, the system itself is also susceptible to random disturbances.

Thus, with the aim of optimal control, it is necessary to estimate the state variables either by a knowingly small or by a large number of measurements, which are themselves inaccurate and are functions of state variables. Using the results of measurements clogged with noise with linear combinations of variable parameters, an assessment of their state can be obtained using a filter consisting of a model of the original system and a feedback signal proportional to the difference between the actual value of the measurement result and its estimate [2].

II. METHODOLOGY

The combination of an optimal filter and an optimal deterministic controller is a feedback controller that is optimal in the sense of the ensemble mean for a linear problem with a quadratic functional and additive Gaussian white noise. In this case, the principle of stochastic equivalence, or the separability theorem [3], is valid.

The optimal control law is found as a result of minimizing the quadratic functional averaged over the ensemble:

$$J = E\left[\frac{1}{2}\lim\frac{1}{T}\int_{t}^{t+T}\left(x^{T}Qx + U^{T}LU\right)dt\right], \tag{1}$$

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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where E – expectation symbol.
The expected value of J is a less accurate measure of system quality. Nevertheless, it is a convenient measure that can serve as a good compromise solution to the task at hand.
Under the assumption of ergodicity, the functional from equation (1) will be rewritten as:

$$J = \frac{1}{2} E\left[E\left(x^T Qx + U^T LU\right)\right] \tag{2}$$

or

$$J = \frac{1}{2} T_2 \left[QX + G^T LGx\right], \tag{3}$$

where x is the deviation of the process state from the desired one; T2 is the trace of the matrix, i.e. the sum of the elements of the main diagonal; X - stationary covariance matrix of state variables; G is the required optimal feedback matrix;

$$U^* = -L^{-1} B^T Px = Gx\,. \tag{4}$$

Here: U^* – optimal control; P(t) – the coefficients found by integrating the Riccati equations in reverse time:

$$\dot{P} = -PA - A^T P - Q + PBL^{-1}BP\,. \tag{5}$$

The result of minimizing the functional is well known from the literature [4] and includes control:
- based on the dynamics of the disturbance vector or on the basis of its assessment;
- with feedback based on state variables or their estimates.

Note that the solution for the matrix G does not depend on whether the system is considered as random or as deterministic with initial conditions determined either by the covariance matrix for a random system or by the values of all state variables at the initial time for a deterministic system [5].
In general, the given part of the servo system with wave channels is described:

$$\text{equation of state } \dot{x} = f(x, U, V, t); \tag{6}$$

$$\text{observation equation } y = g(x, t) + w, \tag{7}$$

where $x = [x_1 \vdots x_2 \vdots x_3]^T - (n_1 + n_2 + n_3)$ – dimensional state vector whose components are: (x_1-n_1) – dimensional vector of the state of the controlled part; (x_2-n_2) – dimensional disturbance vector (environmental conditions); (x_3-n_0) – dimensional vector of the reference signal (uncontrollable part of the system) - the target.

III. SYSTEM ANALYSIS

The rational composition of the state vector is determined taking into account the requirements for a tracking system with wave channels and information about the results of observations. So, the elements of the vector x_1 there can be angular positions, speeds and accelerations of the mechanical, electrical and optical axes of the antenna (equisignal zone - EZ); frequency or phase and their derivatives. In this case, the elements of the vector x3 will be the reference values of the coordinates and their derivatives corresponding to the vector x_1. Behind the vector x_3 tracking is in progress. Vector x_1 characterizes the behavior of the external environment - perturbations acting on a given part of the tracking system with wave channels. The elements of the vector y are the measured components depending on x_1 and x_3. Vektor x_1 characterizes the controlled part of the system, and x_3 – uncontrollable - the goal [6]. The vector w reflects the measurement noise. When measuring a reference signal, these are receiver noise, angular and amplitude target noise, signal fading and other radio interference. The control vector U is formed by the controller, which ensures the optimization of the given criterion of the quality of the system functioning [7].
Vector functions f(·) and g(·) in general, nonlinear, time-varying. The exact form and values of the parameters of the above equations depend on the intended purpose of the tracking system with wave channels.

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International Journal of Advanced Research in Science, Engineering and Technology

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A tracking system with wave channels works as follows: the assessor issues an estimate $\hat{x} = \left[\hat{x}_1 \vdots \hat{x}_2 \vdots \hat{x}_3 \right]^T$ the actual state of a given part of the system x, derived from all interfered-with measurements y up to the time of estimation. This estimate is then used in the regulator when calculating the control signal U acting on the controlled part of the system so as to minimize the adopted criterion. It is assumed that the equations of the estimator and controller can be obtained independently. This assumption can lead to a solution that is not strictly optimal, since servo systems with wave channels are nonlinear systems. However, it allows one to obtain a numerically realizable algorithm leading to quite acceptable results [8].

It is also assumed that it is possible to measure only certain combinations of the state components of a given part of the tracking system with wave channels, which, moreover, are clogged with additive noise. Information on x_3 is contained in a tracking system with wave channels, as a rule, in a narrow-band (generally random) signal. At the output of the receiving device we have:

$$y_3 = g_3(x_3, w_3, t)\,. \tag{8}$$

To get an estimate $\hat{x}_3$ signal x_3 at present, optimal meters (discriminators) with an evaluation device are widely used [9].

Let the linearized model of the servo system with wave channels be described by the equations:

$$\dot{X}_1 = Ax_1 + BU + CV\,, \tag{9}$$

$$y_1 = D_1 x_1 + w_1\,, \tag{10}$$

where (x_1-n) – dimensional state vector; (U-r) – dimensional control vector; (V-s) – dimensional disturbance vector; (y_1-m_1) – dimensional vector of measurements associated with x_1; (w_1-m_1) – dimensional vector of measurement noise.

The disturbing environmental conditions are described using the following equations [9]:

$$\dot{x}_2 = Ex_2 + Fn\,, \tag{11}$$

$$V = Hx_2\,, \tag{12}$$

$$y_2 = D_2 x_2 + w_2\,, \tag{13}$$

where (x_2-g) – dimensional disturbance vector; n(t) – zero mean white noise vector; (y_2-m_2) – dimensional vector of measurements associated with x_2; (w_2-m_2) – dimensional vector of measurement noise.

The behavior of the target is simulated in a similar way - the vector x_3. In equations (9) - (13), the values A, B, C, D1, D2, E, F and H are matrices whose dimensions are determined by the dimensions of the corresponding vectors. Relations (11) - (13) are a model of random perturbations of functions other than white noise. The output signal of a stationary first-order Gaussian Markov process has an exponential correlation function. This indicates a way to investigate problems in which the input noise is random but not white. Using shaping filters of the first or higher order (stationary or non-stationary) with white noise at the input, we obtain time-correlated or colored noise as output signals, i.e. Gaussian Markov process.

Almost any correlation function of noise for practical purposes can be approximated fairly well by an appropriate choice of coefficients in the shaping filter. The state of the entire system in this case will be determined by the extended state vector $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ or (given the purpose) $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. The extended state vector is also a Gaussian Markov process.

The state of a system with an extended state space, including both the object (process) itself and the external influence, is described in this case using the following equations [10]:

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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$$x=\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\left[\frac{A}{O}\,\Big|\,\frac{CH}{E}\right]x+\left[\frac{B}{O}\,\Big|\,\frac{O}{O}\right]\begin{bmatrix}U\\ O\end{bmatrix}+\left[\frac{O}{F}\,\Big|\,\frac{O}{O}\right]\begin{bmatrix}n\\ o\end{bmatrix},$$

$$y=\left[\frac{D_1}{O}\,\Big|\,\frac{O}{D_2}\right], \tag{14}$$

$$w=\begin{bmatrix}w_1\\ w_2\end{bmatrix};\; y=\begin{bmatrix}y_1\\ y_2\end{bmatrix}. \tag{15}$$

Equations (14), (15) can be rewritten as:

$$\dot{X}=\tilde{A}x+\tilde{B}U+\tilde{F}n, \tag{16}$$

$$y=\tilde{D}x+w. \tag{17}$$

In this case, the process x and the noise w are assumed to be independent.
The optimal linear equation with a quadratic functional for such a system has the form:

$$u^*=Gx=[G_1 \vdots G_2]\begin{bmatrix}\hat{x}_1\\ \hat{x}_2\end{bmatrix}. \tag{18}$$

The equations defining the structure and characteristics of the optimal filter can be obtained in various ways. Since their derivation is rather cumbersome, we will restrict ourselves only to a discussion of the final results. The structure of an optimal least squares filter is described by a vector differential equation:

$$\dot{\hat{x}}=\begin{bmatrix}\dot{\hat{x}}_1\\ \dot{\hat{x}}_2\end{bmatrix}=\tilde{A}\hat{X}+\tilde{B}U+\tilde{K}(y-\tilde{y}),\; \hat{X}(t_0)=0, \tag{19}$$

where $\hat{X}$ – appraisal $\hat{x}$; $\tilde{K}$ – an appropriately calculated matrix.

Difference $(y-\hat{y})$ can be physically interpreted as the difference between the observed and predicted values of y obtained by reconstructing the state variables x. By appropriately choosing K, the recovery error $(x-\hat{x})$ can be made zero for arbitrary states of the model of equation (19). Vector initial value $\hat{X}(t_0)$ in equation (19) is usually assumed to be zero. In equation (19), the matrix $\tilde{K}$ is determined from the relation:

$$\tilde{K}(t)=\begin{bmatrix}K_1\\ K_2\end{bmatrix}=\Delta X(t)\tilde{D}^T w, \tag{20}$$

where dimension K_1-nxm, and K_2-gxm, ΔX and $w\delta(\tau)=E\left[w(t)w^T(t+\tau)\right]$ – square covariance matrices of state estimation errors and measurement noise, respectively.

Proportionality matrix $\tilde{K}$ actually characterizes the relationship between the uncertainty of the state ΔX and measurement uncertainty w [11].

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International Journal of Advanced Research in Science, Engineering and Technology
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Covariance matrix ΔX(t) defined as:

$$\Delta X(t) = E\left[\Delta X(t)\Delta X^T(t)\right] == E\left\{\left[\hat{X}(t) - X(t)\right]\left[\hat{X}(t) - X(t)\right]^T\right\} == E\begin{bmatrix} (\hat{x}_1 - x_1)^2 \ldots (\hat{x}_1 - x_1)(\hat{x}_n - x_n) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ (\hat{x}_n - x_n)(\hat{x}_1 - x_1)\ldots(\hat{x}_n - x_n)^2 \end{bmatrix}. \quad (21)$$

This assumes that the expectation operator is applied to each element of the matrix. The diagonal elements of the matrix are the variances of the vector components. Elements outside the diagonal are mixed elements of the second order. The matrix ΔX(t) – symmetrical, consisting of n(n+1)/2 various elements. Thus, the elements of this matrix are the variances and mutual variances of filtering errors of the expected components of the vector X(t). Matrix element values ΔX(t) can be determined by solving a nonlinear equation like the matrix Riccati equation:

$$\Delta X(t) = \tilde{A}\Delta X(t) + \Delta X(t)\tilde{A}^T + \tilde{F}N\tilde{F}^T - \Delta X(t)\tilde{D}^T w^1 \tilde{D}\Delta X(t),$$
$$\Delta X(t_0) = \Delta X_0, \quad (22)$$

where $N\delta(\tau) = E\left[n(t)n^T(t+\tau)\right]$.

Equation (22) characterizes the change in filtering errors over time. The first two terms on the right-hand side of equation (22) represent the change in the covariance matrix of reconstruction errors associated with the dynamics of the system. The third term is the change in the covariance matrix of reconstruction errors caused by the disturbance V acting on the system. And, finally, the last term characterizes the change in the covariance matrix of reconstruction errors as a result of the measurements. It depends on the choice of the matrix gain $\tilde{K}$.

To solve equation (22), it is necessary to set the initial value $\Delta X(t_0)$ error variance matrices. If at the moment $t=t_0$ process $\hat{X}(t_0)$ at the filter output is equal to zero, then, as can be seen from equation (21), the matrix $\Delta X(t_0)$ is equal to the variance matrix of the components of the filtered process X(t) in the moment $t=t_0$. $\Delta X(t_0) = E\{[-X(t_0)][-X^T(t_0)]\}$ [6].

The structure of a complete system, consisting directly of the controlled process, the external environment, an evaluator of states (Kalman-Bucy filter) and optimal feedback, is shown in

ISSN: 2350-0328
International Journal of Advanced Research in Science, Engineering and Technology
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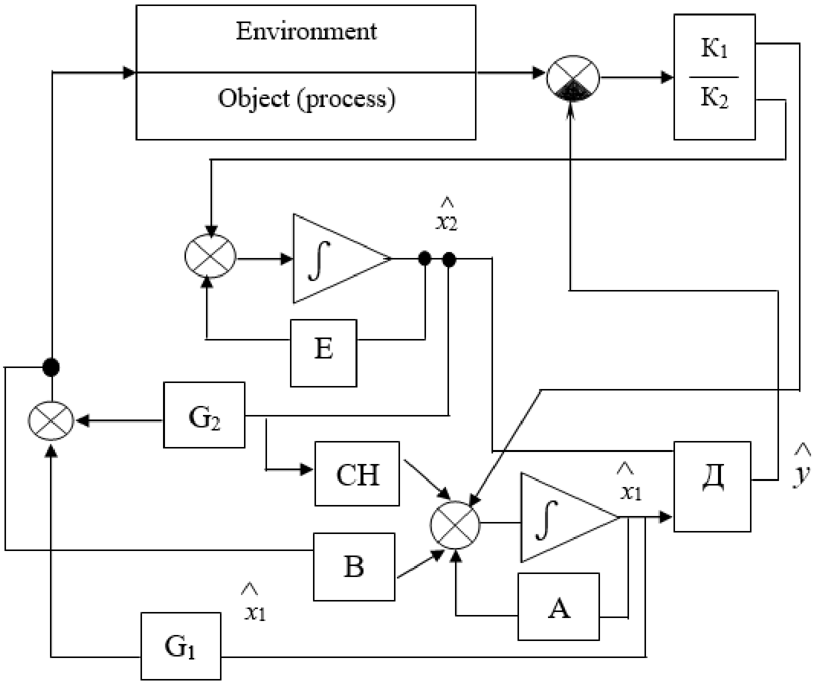

Figure: 1. The structure of the complete control system.

Fig. 1. It is shown for the case when the state x_2 the external environment is not measured.
Thus, to determine the control, it is necessary to solve two ordinary differential equations: equation (22) for the matrix ΔX for a given initial condition and the equation

$$U^* = -L^{-1}B^T PX = GX \tag{23}$$

for the matrix P under a given final condition. Here, the coefficients P(t) are found by integrating the Riccati equations in reverse time [6]:

$$\dot{P} = -PA - A^T P - Q + PBL^{-1}BP \ . \tag{24}$$

The key to the practical design of the Kalman filter lies in the correct choice of the values of the elements of the covariance matrices w and N. Typically, the bandwidth of the filter depends quantitatively on the ratio $\|w\|\,\|N\|^{-1}$ [7]. The larger it is, the smaller the filter bandwidth. This ratio determines the trade-off between the state recovery rate and the resistance to observation noise. Shifting the observation poles to either half of the complex plane results in an increase in bandwidth and hence in recovery speed.

IV. RESULTS

In practice, the matrices N and w are chosen in such a way that the poles of the estimator and controller are at a distance of approximately one order of magnitude from the origin. It is uneconomical to have very high speed regulation when the recovery process is slow, and vice versa. In particular, when the observation noise is much larger than the excitation noise, the observation poles are relatively close to the origin and the reconstruction process is slow. If we now achieve the speed of the regulator somewhat higher than that of the observer, then one should expect that the regulator will be restrained by the observer. A further increase in the speed of the controller will only increase the average value of the square of the input variable without changing the average value of the square of the control error. On the other hand, when the observation noise is small, the allowable mean value of the square of the input variable becomes the limiting

factor in the system. This limits the speed of the regulator and it becomes impractical to choose an observer with a very high speed of response, even if the noise characteristics allow it.
In conclusion, it should be noted that, in contrast to the optimal controller, the optimal observer can be realized in the RMV, since Eq. (22) is a differential equation with given initial conditions, while in the optimal control problem it is necessary to solve the Riccati equation under given final conditions in the reverse time [12].

REFERENCES

1. Gulyamov Sh.M., Yusupbekov A.N., Rashidov Yu.R., Ataullaev A.O. Methods of calculating controls in a tracking system with wave channels, taking into account complex interference and signal conditions // Journal "Industrial ACS and Controllers". - Moscow. 2014. - №1. – p. 24-30.
2. Zamolodchikov V.N., Chilikin V.M. Synthesis of discriminators and filters of radiotechnical tracking systems // Educational manual on the course "Radioautomatics". Ed. by Yu.A. Evsikov. - M.: Izd-vo MPEI, 1993. - 115 p.
3. Grigoriev V.V., Bystrov S.V., Boyekov V.I., Boltunov G.I., Korovyakov A.N., Mansurova O.K., Pershin I.M. Design of regulators for stochastic systems and objects with uncertain parameters. – SPb: NIU ITMO, 2013. -172 p.
4. Bakulev P.A. Radar systems. Textbook for universities, "Radio Engineering", 2004. – 323 p.
5. Gulyamov Sh.M., Yusupbekov A.N., Ataullaev A.O., Abdullayeva K.R. Some issues of synthesis of tracking systems with wave channels // Journal "Automation and modern technologies". - Moscow. 2016. - № 1. – p. 37-42.
6. Yusupbekov A.N., Ataullaev A.O. Synthesis of the automatic gain control system of the tracking system. Journal "MOUNTAIN BULLETIN of Uzbekistan" - Navoi. 2012.- No 2. - p. 64-66.
7. Yusupbekov A.N., Ataullaev A.O., Artykov S.Z. Synthesis of a tracking system with wave channels taking into account measurement noises and disturbances on the system // "Ishlab chikarishdagi machinelar va mechanic zhikhozlar innovation technologari, yutuq va vasifalar" Republic of Ilmii va ilmiy-tekhnik anzhumani materiallari. – Fergana, 2013. – p. 28-30.
8. Yusupbekov A.N., Temerbekova B.M. , Ataullaev A.O. Application of artificial neural networks to the assessment of the parameters of the state of a mobile object based on the results of measurements // International Scientific and Technical Journal "Chemical Technology. Control and management". - Tashkent, 2012. - №3. – p. 38-39.
9. Yusupbekov A.N., Ataullaev A.O. Synthesis of Servo System With Wavy Channels Based Measurements and Noise Disturbance in the System// «Гидротехника иншоотларининг самарадорлигини, ишончлилиги ва хавфсизлигини ошириш» Республика илмий-амалий конференция материаллари. - Тошкент, 2012. – с.104÷107.
10. Yusupbekov A.N., Ataullaev A.O., Ergashev F.A., Tekeyev M.E. Management of Dynamic Objects Using Neural Network Technology// Transactions of the International Scientific Conference «Modern Problems of Applied Mathematics and Information Technologies – AL – KHOREZMIY – 2014». - Samarqand, 2014. – pp. 53-56.
11. Yusupbekov A.N., Gulyamov Sh.M., Ataullayev A.O., Shamsutdinova V.H. Synthesis of the Sepvo System with Wave Channels// Journal of the Technical University of Gabrovo. - Bulgaria, Gabrovo, Volume 51' 2015. - pp. 56-60.
12. Yusypbekov A.N., Ataullaev A.O., Artikov S.Z., Temerbekova B.M. Synthesis of servo system with wavy channels-based measurements and noise disturbance in the system// Transactions of the international scientific conference «Modern problems of applied mathematics and information technologies – AL – KHOREZMIY 2014» - Samarqand, 2014. – pp. 51-53.