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Comparison of Measures of Parameter of the Model $X=\mu+\varepsilon$ Based On Pythagorean Means

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ABSTRACT: Four formulations of average namely Arithmetic-Geometric Mean (abbreviated as *AGM*), Arithmetic-Harmonic Mean (abbreviated as *AHM*), Geometric-Harmonic Mean (abbreviated as *GHM*) and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) have recently been derived from the three Pythagorean means namely Arithmetic Mean (*AM*), Geometric Mean (*GM*) and Harmonic Mean (*HM*). Each of these four formulations has been found to be a measure of parameter μ of the model $X = \mu + \varepsilon$ in addition to *AM*, *GM* & *HM*. This paper is a description on the comparison of these measures of parameter μ of this model.

KEYWORDS: *AM*, *GM*, *HM*, *AGM*, *AHM*, *GHM*, *AGHM*, Observed data, Measure of parameter.

I. INTRODUCTION

Several research had already been done on developing definitions / formulations of average [1 , 2], a basic concept used in developing most of the measures used in analysis of data. Pythagoras [3], the pioneer of researchers in this area, constructed three definitions / formulations of average namely Arithmetic Mean, Geometric Mean & Harmonic Mean which are called Pythagorean means [4 , 5 , 14 , 18]. A lot of definitions / formulations have already been developed among which some are arithmetic mean. geometric mean, harmonic mean, quadratic mean, cubic mean, square root mean, cube root mean, general *p* mean and many others [6 , 7 , 8 , 9 , 10 , 11 , 12 , 13 , 14 , 15 , 16 , 17, 18 , 19]. Kolmogorov [20] formulated one generalized definition of average namely Generalized *f* - Mean. [7 , 8]. It has been shown that the definitions/formulations of the existing means and also of some new means can be derived from this Generalized *f* - Mean [9 , 10]. In an study, Chakrabarty formulated one generalized definition of average namely Generalized f_H – Mean [11]. In another study, Chakrabarty formulated another generalized definition of average namely Generalized f_G – Mean [12 , 13] and developed one general method of defining average [15, 16 , 17] as well as the different formulations of average from the first principles [19].
In many real situations, observed numerical data

$$x_1 , x_2 , \ldots\ldots , x_N$$

are found to be composed of a single parameter μ and corresponding chance / random errors

$$\varepsilon_1 , \varepsilon_2 , \ldots\ldots , \varepsilon_N$$

i.e. the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots , N) \tag{1.1}$$

[21 , 22 , 23 , 24 , 25 , 26 , 27, 28 , 29].
The existing methods of estimation of the parameter μ namely least squares method, maximum likelihood method, minimum variance unbiased method, method of moment and method of minimum chi-square, [31 – 52] cannot provide appropriate value of the parameter μ [21 , 22 , 23]. In some recent studies, some methods have been developed for determining the value of parameter from observed data containing the parameter itself and random error [21 ,22 , 23 , 24 , 25 , 26, 27 , 28 , 29 , 30 , 53 ,54 , 55 , 56, 57 , 58 , 59 , 60]. The methods, developed in this studies, for determining the appropriate value of the parameter from observed data containing the parameter itself and random error involve huge computational tasks. Moreover, a finite set of observed data may not yield the appropriate value of the parameter in many situations while the number of observations required in the methods may be too large for obtaining the appropriate value of the parameter. However, the appropriate value of the parameter is not perfectly attainable in practical situation. What one can expect is to obtain that value which is more and more close to the appropriate value of the parameter. In order to obtain such value of parameter, four methods have already been developed which involves lesser computational tasks than those involved in the earlier methods as well as which can be applicable in the case of finite set of data [61 , 62 , 63 , 64]. The methods developed are based on the concepts of Arithmetic-Geometric Mean

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(abbreviated as *AGM*) [61 , 62, 67 , 68], Arithmetic-Harmonic Mean (abbreviated as *AHM*) [63], Geometric-Harmonic Mean (abbreviated as *GHM*) [64] and Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) [65, 66] respectively. Each of these four formulations namely *AGM*, *AHM*, *GHM* & *AGHM* has been found to be a measure of parameter μ of the model $X = \mu + \varepsilon$ in addition to *AM*, *GM* & *HM*. This paper is a description on the comparison of these measures of parameter μ of this model.

II. FORMULATIONS OF *AGM, AHM, GHM & AGHM*

Let a_0 , g_0 & h_0 be respectively the Arithmetic Mean (*AM*), the Geometric Mean (*GM*) & the Harmonic Mean (*HM*) of N positive numbers or values or observations (not all equal or identical)

$$x_1 , x_2 , \ldots\ldots , x_N$$

all of which are not equal i.e.

$$a_0 = AM\,(x_1 , x_2 , \ldots\ldots , x_N) = \frac{1}{N}\sum_{i=1}^{N} x_i ,$$
$$g_0 = GM\,(x_1 , x_2 , \ldots\ldots , x_N) = \left(\prod_{i=1}^{N} x_i\right)^{1/N}$$
$$\&\ h_0 = HM\,(x_1 , x_2 , \ldots\ldots , x_N) = \left(\frac{1}{N}\sum_{i=1}^{N} x_i^{-1}\right)^{-1}$$

which satisfy the inequality [4 , 5] namely

$$AM > GM > HM \quad \text{i.e.} \quad h_0 < g_0 < a_0 \qquad (2.1)$$

ARITHMETIC-GEOMETRIC (AGM)

The two sequences $\{a_n\}$ & $\{g_n\}$ respectively defined by

$$a_{n+1} = ½\, a_n + g_n) ,$$
$$\& \quad g_{n+1} = (a_n\, g_n)^{1/2}$$

& the square root takes the principal value,
converge to a common point M_{AG} as n approaches infinity.
This common converging point M_{AG} can be termed / named / regarded as the Arithmetic-Geometric Mean (abbreviated as *AGM*) of the *N* numbers (or values or observations) $x_1 , x_2 , \ldots\ldots , x_N$ [61 , 62, 66 , 67 , 68].

ARITHMETIC-HARMONIC MEAN (AHM)

Let $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{h'_n = h'_n(a_0 , h_0)\}$ be two sequences defined by

$$a'_{n+1} = ½(\, a'_n + h'_n)$$
$$\& \quad h'_{n+1} = ½(\, {a'_n}^{-1} + {h'_n}^{-1})\}^{-1}$$

respectively.
Then, the two sequences $\{a'_n = a'_n(a_0 , h_0)\}$ & $\{h'_n = h'_n(a_0 , h_0)\}$ converge to common point M_{AH} as n approaches infinity.
This common converging point M_{AH} can be termed / named / regarded as the Arithmetic-Harmonic Mean (abbreviated as *AHM*) of the *N* numbers (or values or observations) $x_1 , x_2 , \ldots\ldots , x_N$ [63 , 66].

GEOMETRIC-HARMONIC MEAN (GHM)

The two sequences $\{g''_n\}$ & $\{h''_n\}$ defined respectively by

$$g''_{n+1} = (\, g''_n \,.\, h''_n\,)^{1/2}$$
$$\& \quad h''_{n+1} = \{½(\, {g''_n}^{-1} + {h''_n}^{-1}\,)\}^{-1}$$

where the square root takes the principal value,
converge to common point M_{GH} as n approaches infinity. This common converging point M_{GH} can be termed / named / regarded as the Geometric-Harmonic Mean (abbreviated as *GHM*) of the *N* numbers (or values or observations) x_1 , $x_2 , \ldots\ldots , x_N$ [64 , 66].

ARITHMETIC-GEOMETRIC-HARMONIC MEAN (AGHM)

The three sequences $\{a'''_n\}$, $\{ g'''_n \}$ & $\{ h'''_n \}$ defined respectively by

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$$a'''_n = 1/3\,(a'''_{n-1} + g'''_{n-1} + h'''_{n-1}) \;,$$
$$g'''_n = (a'''_{n-1}\, g'''_{n-1}\, h'''_{n-1})^{1/3}$$
$$\&\; h'''_n = \{1/3\,({a'''_{n-1}}^{-1} + {g'''_{n-1}}^{-1} + {h'''_{n-1}}^{-1})\}^{-1}$$

converges to a common limit M_{AGH} as n approaches infinity.
This common converging point M_{AGH} can be termed / named / regarded as the Arithmetic-Geometric-Harmonic Mean (abbreviated as *AGHM*) of the *N* numbers (or values or observations) $x_1, x_2, \ldots\ldots, x_N$ [65 , 66].

III. MEASURES OF VALUE OF PARAMETER μ

If the observations

$$x_1, x_2, \ldots\ldots, x_N$$

(which are strictly positive and not all identical) are composed of some parameter μ and random errors then the observations can be expressed as

$$x_i = \mu + \varepsilon_i \quad , \quad (i = 1, 2, \ldots\ldots, N)$$

where

$$\varepsilon_1, \varepsilon_2, \ldots\ldots, \varepsilon_N$$

are the random errors associated to

$$x_1, x_2, \ldots\ldots, x_N$$

respectively which assume positive real values and negative real values in random order.

The parameter μ, in this case, can be interpreted as the central tendency of the observations $x_1, x_2, \ldots\ldots, x_N$.

Then, each of $AM\,(x_1, x_2, \ldots\ldots, x_N)$, $GM\,(x_1, x_2, \ldots\ldots, x_N)$ & $HM\,(x_1, x_2, \ldots\ldots, x_N)$ becomes closer and closer to μ as the data size *N* becomes larger and larger.
This implies that each of $AM\,(x_1, x_2, \ldots\ldots, x_N)$, $GM\,(x_1, x_2, \ldots\ldots, x_N)$ & $HM\,(x_1, x_2, \ldots\ldots, x_N)$ can be regarded as a measure of the value of the parameter μ.

Let

$$a_0 = AM\,(x_1, x_2, \ldots\ldots, x_N) = \frac{1}{N}\sum_{i=1}^{N} x_i \;,$$
$$g_0 = GM\,(x_1, x_2, \ldots\ldots, x_N) = \left(\prod_{i=1}^{N} x_i\right)^{1/N}$$
$$\&\; h_0 = HM\,(x_1, x_2, \ldots\ldots, x_N) = \left(\frac{1}{N}\sum_{i=1}^{N} {x_i}^{-1}\right)^{-1}$$

In the case the two sequences $\{a_n\}$ & $\{g_n\}$ defined above, it is established that these two sequences converge to a common point which is very close to μ [61 , 62].
This implies that $AGM\,(x_1, x_2, \ldots\ldots, x_N)$ can be a measure of the value of the parameter μ.

Similarly in the two sequences $\{a'_n\}$ & $\{h'_n\}$ defined above, it is also established that these two sequences converge to a common point which is very close to μ [63].
This implies that $AHM\,(x_1, x_2, \ldots\ldots, x_N)$ can also be a measure of the value of the parameter μ.

Again in the two sequences $\{g''_n\}$ & $\{h''_n\}$ defined above, it is proved that these two sequences converge to a common point which is very close to μ [64].
This implies that $GHM\,(x_1, x_2, \ldots\ldots, x_N)$ can also be a measure of the value of the parameter μ.

Similarly in the three sequences $\{a'''_n\}$, $\{g'''_n\}$ & $\{h'''_n\}$ defined above, it is also proved that these three sequences converge to a common point which is very close to μ [65 , 66].
This implies that $AGHM\,(x_1, x_2, \ldots\ldots, x_N)$ can also be a measure of the value of the parameter μ.

Note: Let $y_1, y_2, \ldots\ldots, y_N$
be such that $y_i = (x_i - a)/h$ where *a* & *h* are non-zero real.
Then the model (1.1) comes down to be

$$y_i = \mu' + \varepsilon_i' \;, \quad (i = 1, 2, \ldots\ldots, N) \qquad (3.1)$$

where $\mu^{/} = (\mu - a) / h$ & $\varepsilon_i^{/} = \varepsilon_i / h$

IV. COMPARISON OF THE MEASURES

Inequality (2.1) implies that

$AM\,(x_1 , x_2 , \ldots\ldots , x_N) > GM\,(x_1 , x_2 , \ldots\ldots , x_N) > HM\,(x_1 , x_2 , \ldots\ldots , x_N)$

Since the two sequences $\{a_n\}$ & $\{g_n\}$ converge to a common point lying between $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $GM\,(x_1 , x_2 , \ldots\ldots , x_N)$

Therefore,

$AGM\,(x_1 , x_2 , \ldots\ldots , x_N)$ lies between $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $GM\,(x_1 , x_2 , \ldots\ldots , x_N)$.

Similarly,

$AHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ lies between $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$,

$GHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ lies between $GM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$

& $AGHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ lies between $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$.

Thus, each of these four measures lies between $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$.

Thus for a finite set of N observations $x_1 , x_2 , \ldots\ldots , x_N$, the following results are consequences of this fact:

(1) If $AM\,(x_1 , x_2 , \ldots\ldots , x_N) < \mu$
i.e. if $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ is an underestimate of μ
then the other measures are automatically underestimates of μ.
In this case, by the above inequality, $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$ is closest to μ among the seven measures and hence can be accepted as the most accurate measure of the value of μ.

(2) If $HM\,(x_1 , x_2 , \ldots\ldots , x_N) > \mu$
i.e. if $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$ is an over estimate of μ
then the other measures are automatically overestimates of μ.
In this case, by the above inequality, $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$ is closest to μ among the seven measures and therefore can be accepted as the most accurate measure of the value of μ.

(3) If $AM\,(x_1 , x_2 , \ldots\ldots , x_N) > \mu$ but $GM\,(x_1 , x_2 , \ldots\ldots , x_N) < \mu$
then $AGM\,(x_1 , x_2 , \ldots\ldots , x_N)$ is closest to μ
and therefore can be accepted as the most accurate measure of the value of μ.

(4) If $AM\,(x_1 , x_2 , \ldots\ldots , x_N) > \mu$ but $HM\,(x_1 , x_2 , \ldots\ldots , x_N) < \mu$
then $AGHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $AHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ are more closer to μ than the others.
This implies that the compromising value of $AGHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $AHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ can be accepted as the most accurate value of μ in this case.

(5) If $GM\,(x_1 , x_2 , \ldots\ldots , x_N) > \mu$ but $HM\,(x_1 , x_2 , \ldots\ldots , x_N) > \mu$
then $GHM\,(x_1 , x_2 , \ldots\ldots , x_N)$ is closest to μ among the seven measures
and therefore can be accepted as the most accurate measure of the value of μ in this case.

Note: If it is known whether $AM\,(x_1 , x_2 , \ldots\ldots , x_N)$, $GM\,(x_1 , x_2 , \ldots\ldots , x_N)$ & $HM\,(x_1 , x_2 , \ldots\ldots , x_N)$ are under estimate(s) or overestimate(s) then the compromising value of all the seven measures can be considered to be acceptable as most possible accurate value of μ.

V. NUMERICAL EXAMPLE: APPLICATION TO NUMERICAL DATA

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013. The objective here is to evaluate the central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati

A. Central Tendency of Annual Maximum of Surface Air Temperature at Guwahati

From the observed data on annual maximum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, the following values (in Degree Celsius) had been obtained [61 , 62 , 63 , 64 , 65 , 66].

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$$
\begin{aligned}
AM &= 37.2093023255814\ , \\
GM &= 37.1922871485760\ , \\
HM &= 37.17539890356262\ , \\
AGM &= 37.200794250670693716568240158 13\ , \\
AHM &= 37.188111479222283218438295127449\ , \\
GHM &= 37.18384158788008150488383097978 6 \\
\&\ AGHM &= 37.19232688378569045281501129744 1
\end{aligned}
$$

Now, the compromising value of most of these seven is 37.1.
Hence, 37.1 Degree Celsius can be considered as the value of central tendency of Annual Maximum of Surface Air Temperature at Guwahati.

B. Central Tendency of Annual Minimum of Surface Air Temperature at Guwahati

From the observed data on annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, the following values (in Degree Celsius) had been obtained [61 , 62 , 63 , 64 , 65 , 66].
:

$$
\begin{aligned}
AM &= 7.36341463414634146341463414634 15\ , \\
GM &= 7.25971761945761856087096163512 97\ , \\
HM &= 7.15439338028235252098497447075 69\ , \\
AGM &= 7.31147420703016646412362218358 25\ , \\
AHM &= 7.25815161833994661021742795089 2\ , \\
GHM &= 7.20676689513737000737937277008 02\ , \\
\&\ AGHM &= 7.25867355712886575553931587745 38
\end{aligned}
$$

In this case, the compromising value of most of these seven is 7.2.
Hence, 7.2 Degree Celsius can be considered as the value of central tendency of Annual Minimum of Surface Air Temperature at Guwahati.

VI. CONCLUSION

Each of the four formulations *AGM*, *AHM*, *GHM* & *AGHM* can be accepted as a measure of parameter μ of the model described by equation (1.1) in addition to *AM*, *GM* & *HM*.

Each of the three measures namely *AM*, *GM*, & *HM* becomes closer and closer to the value of parameter μ as the data size becomes larger and larger. On the other hand, each of the four measures namely *AGM*, *AHM*, *GHM* & *AGHM* moves to be closer to the value of parameter μ as the term n of the respective sequence increases where it is not necessary that the data size is to be large. Therefore, for small data set, the four measures namely *AGM*, *AHM*, *GHM* & *AGHM* can yield the value which is more close to the actual value of parameter μ than that yielded by *AM*, *GM*, & *HM*. However, if data size is large, the three measures namely *AM*, *GM*, & *HM* can yield more accurate value of the parameter. In this case also *AGM*, *AHM*, *GHM* & *AGHM* can also be used for the purpose.

On the whole, it can be concluded that for small sample *AGM*, *AHM*, *GHM* & *AGHM* can be treated as alternatives of *AM*, *GM*, & *HM* in order to obtain reasonable estimate of the parameter μ of the model described by equation (1.1).

It is to be noted that each of the above formulations for determining the value of parameter μ of the model described by equation (1.1) is valid only when the observed values

are strictly positive.
If the observed values are not all positive then the change of origin and scale can be applied to make the observed values strictly positive so that the transformed values follow the model(3.1). Then the value of the parameter the model (3.1) can be computed by the above formulations. Finally, the value of the parameter of the model (1.1)can be obtained from this value.

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Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Senior Diploma (in Guitar) from Prayag Sangeet Samiti in 2019 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

(Dr. Dhritikesh Chakrabarty, Right in the first row, attending World Congress Engineers in Imperial College, London, July 02 - 04, 2014)

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