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Algorithms for the synthesis of stabilizing state controllers for discrete objects based on linear matrix inequalities

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ABSTRACT: This article presents the algorithms for the synthesis of stabilizing state controllers for discrete objects based on linear matrix inequalities. An approach to solving the problem of synthesis of stabilizing state controllers for discrete objects based on the use of linear matrix inequalities is proposed. Vital and sufficient conditions for the existence of linear-quadratic discrete state controllers are formulated. The proposed algorithms make it possible to ensure that the requirements for the control accuracy are met, that is, the steady-state values of the controlled variables fall within the specified tolerances when the system is acted upon by bounded external disturbances.

KEY WORDS: discrete object, state controller, synthesis algorithms, stabilizing controller, linear matrix inequalities

I. INTRODUCTION

The problem of the synthesis of controllers that guarantee the fulfillment of the requirements for the control accuracy, which the steady-state values of the controlled variables fall within the given tolerances, when the system is acted on by bounded external disturbances. It is one of the central ones in the theory and practice of automatic control. With the inclusion of digital computers in the control loop, an urgent problem arises of selecting discrete control laws for continuous dynamic objects [1-8].

II. FORMULATION OF THE PROBLEM

We will consider a linear stationary discrete object described in the state space by a difference equation of the form:

$$x_{t+1} = Ax_t + Bu_t \, , \tag{1}$$

where $x_t \in \Re^{n_x}$ – object state, $u_t \in \Re^{n_u}$ – control, $A \in \Re^{n_x \times n_x}$ and $B \in \Re^{n_x \times n_u}$ – given matrices.

If all the state vector variables x_t can be measured, then for the controlled plant (1) the problem of a stabilizing controller by state arises, which is reduced to select a control law from the class of linear feedbacks of the form:

$$u_t = \Theta x_t \, , \tag{2}$$

where $\Theta \in \Re^{n_u + n_x}$ – matrix of controller parameters at which the state $x_t = 0$. The closed-loop system (1), (2) will be:

$$x_{t+1} = A_c x_t, \; A_c = A + B\Theta, \tag{3}$$

and it is asymptotically stable in the sense of Lyapunov [1,2].

III. SOLUTION OF THE TASK

As one of the possible ways to solve the problem of stabilization of an unstable plant by state, one can consider a path based on the use of linear matrix inequalities. The synthesis of stabilizing controllers is based on the Lyapunov method. Specifically, the fact that the stabilization of a discrete object is equivalent to the existence of a quadratic function V_t

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such that any trajectory of the closed-loop system satisfies the inequality $\Delta V_t < 0$. According to this approach, the condition for the stabilization of a discrete object (1) is equivalent to the solution of the Lyapunov inequality:

$$A_c^T X A_c - X < 0,$$

It is a unknown matrix $X = X^T > 0$ and the matrix of the controller parameters Θ [6,9]:

$$(A + B\Theta)^T X (A + B\Theta) - X < 0. \quad (4)$$

The resulting inequality is not linear with respect to unknown matrices $X = X^T > 0$ and Θ, therefore, it should be represented as a linear matrix inequality. Consider two ways of solving the problem.
The first way of solving the problem of synthesis of stabilizing controllers is to represent inequality (4) in the form of a matrix inequality of some special form:

$$\Psi + P^T \Theta^T Q + Q^T \Theta P < 0, \quad (5)$$

where P and Q – matrices of the corresponding orders depending on the initial data of the problem, $\Psi = \Psi^T$ – a matrix that will contain an unknown matrix $X = X^T > 0$.
Conditions for the solving of inequality (5) with respect to an unknown matrix Θ can be represented as following:
Given a symmetric matrix $\Psi = \Psi^T \in R^{n\times n}$ and two matrices $P \in R^{l\times n}$ and $Q \in R^{k\times n}$, moreover $rank P = r_P < n$ and $rank Q = r_Q < n$. Linear matrix inequality (5) solvable with respect to the matrix $\Theta \in R^{k\times l}$ if and only if the inequalities are solvable:

$$W_P^T \Psi W_P < 0, \; W_Q^T \Psi W_Q < 0, \quad (6)$$

where the columns of the matrix W_P form the basis of the kernel of the matrix P, and the columns of the matrix W_Q form the basis of the kernel of the matrix Q.
Discrete object (1) stabilizes if and only if there is $(n_x \times n_x)$ - matrix $Y = Y^T > 0$, satisfying the linear matrix inequality:

$$W_{B^T}^T (AYA^T - Y) W_{B^T} < 0, \quad (7)$$

where the columns of the matrix W_{B^T} form the basis of the kernel of the matrix B^T. If inequality (7) is solvable with respect to the matrix Y, then the parameters Θ linear state feedback are found as solutions to the linear matrix inequality:

$$\begin{pmatrix} -Y & A + B\Theta \\ (A + B\Theta)^T & -Y^{-1} \end{pmatrix} < 0. \quad (8)$$

According to the discrete version of the Lyapunov theorem [1,2,9,10], the stabilization of the object (1) is equivalent to the existence of the quadratic Lyapunov function $V_t(x_t) = x_t^T X x_t$, Where $X = X^T > 0$, such that along any trajectory of the closed-loop system:

$$\Delta V_t = V_{t+1} - V_t = x_{t+1}^T X x_{t+1} - x_t^T X x_t = x_t^T (A_c^T X A_c - X) x_t < 0.$$

This condition is equivalent to the following matrix inequalities:

$$A_c^T X A_c - X < 0, \qquad X = X^T > 0. \quad (9)$$

Taking into account Schur's lemma [11-13] and the form of the matrix A_c inequalities (9) can be represented as follows:

$$\begin{pmatrix} -X^{-1} & A + B\Theta \\ A^T + \Theta^T B^T & -X \end{pmatrix} < 0. \quad (10)$$

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We will write the last inequality as

$$\begin{pmatrix} -X^{-1} & A \\ A^T & -X \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ \Theta^T B^T & 0 \end{pmatrix} + \begin{pmatrix} 0 & B\Theta \\ 0 & 0 \end{pmatrix} < 0 .$$

Because

$$P^T \Theta^T Q = \begin{pmatrix} 0 & 0 \\ \Theta^T B^T & 0 \end{pmatrix} = \begin{pmatrix} 0 \\ I \end{pmatrix} \Theta^T \begin{pmatrix} B^T & 0 \end{pmatrix},$$

from this, we can obtain a representation of inequality (10) in the form (5), where $\Psi = \begin{pmatrix} -X^{-1} & A \\ A^T & -X \end{pmatrix}$,

$$P = \begin{pmatrix} 0_{n_x \times n_x} & I_{n_x} \end{pmatrix}, \qquad Q = \begin{pmatrix} B^T & 0_{n_u \times n_x} \end{pmatrix}.$$

We will take a note that,

$$rank\ P = n_x < 2n_x,\ rank\ Q = rank\ B^T < 2n_x,$$

then, according to (6), inequality (5) is solvable with respect to the matrix of parameters if and only if the inequalities are solvable:

$$W_P^T \Psi W_P < 0, \quad W_Q^T \Psi W_Q < 0,$$

where the columns of the matrix W_P form the basis of the kernel of the matrix P, and the columns of the matrix W_Q form the basis of the kernel of the matrix Q. It means that matrices of maximum rank W_Q and Q must satisfy the matrix equation $PW_p = 0$ and $QW_Q = 0$, respectively. By direct verification, it may take for such matrices:

$$W_P = \begin{pmatrix} I_{n_x} \\ 0_{n_x \times n_x} \end{pmatrix},\ W_Q = \begin{pmatrix} W_{B^T} & 0_{n_x \times n_x} \\ 0 & I_{n_x} \end{pmatrix},$$

where the columns of the matrix W_{B^T} form the basis of the kernel of the matrix B^T. Matrix W_{B^T} satisfies the matrix equation $B^T W_{B^T} = 0$ and shows the highest rank among all its solutions.

As a result, we can obtain that inequality (5) is solvable if and only if the inequalities

$$\begin{pmatrix} I \\ 0 \end{pmatrix}^T \begin{pmatrix} -X^{-1} & A \\ A^T & -X \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix} < 0. \tag{12}$$

$$\begin{pmatrix} W_{B^T} & 0 \\ 0 & I \end{pmatrix}^T \begin{pmatrix} -X^{-1} & A \\ A^T & -X \end{pmatrix} \begin{pmatrix} W_{B^T} & 0 \\ 0 & I \end{pmatrix} < 0.$$

Because

$$\begin{pmatrix} I \\ 0 \end{pmatrix}^T \begin{pmatrix} -X^{-1} & A \\ A^T & -X \end{pmatrix} \begin{pmatrix} I \\ 0 \end{pmatrix} = -X^{-1} < 0,$$

is always satisfied due to the positive existence of the matrix X, then it remains to check the fulfillment of the second conditions (12), which is equivalent to the inequality:

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$$\begin{pmatrix} -W_{B^T}^T X^{-1} W_{B^T} & W_{B^T}^T \\ A^T W_{B^T} & -X \end{pmatrix} < 0 . \tag{13}$$

By Schur's lemma [11,12] (13) holds if and only if:

$$W_{B^T}^T (AX^{-1}A^T - X^{-1}) W_{B^T} < 0 ,\ X = X^T > 0 .$$

Let us introduce the notation $X^{-1} = Y$, where $Y = Y^T > 0$ and we will obtain matrix inequality (7) linearly respect to the matrix Y.
If the linear matrix inequality (7) is solvable, then one of its possible solutions can be found - the matrix Y. Substituting further Y at (8), we obtain a linear matrix inequality with respect to the matrix Θ, solved which we find the parameters of the linear state feedback.
The second way to solve the problem of stabilization with respect to the state of a discrete object (1) is based on the application of the following statement.
We stabilize the discrete object (1) if and only if there exist $(n_x \times n_x)$ - matrix $Y = Y^T > 0$ and $(n_u \times n_x)$ - matrix Z, satisfying the linear matrix inequality:

$$\begin{pmatrix} -Y & AY + BZ \\ (AY + BZ)^T & -Y \end{pmatrix} < 0 . \tag{14}$$

If inequality (14) is solvable for the matrices Y and Z, then the parameters Θ of linear state feedback are found as $\Theta = ZY^{-1}$.
We stabilize the object (1) if and only if there is a matrix $X = X^T > 0$, satisfying the inequality:

$$A_c^T X A_c - X < 0 .$$

We multiply the resulting inequality on the left and right by the matrix $X^{-1} > 0$ and get:

$$X^{-1} A_c^T X A_c X^{-1} - X^{-1} < 0 ,$$

or, by Schur's lemma, the last inequality is equivalent to the following [11-13]:

$$\begin{pmatrix} -X^{-1} & A_c X^{-1} \\ X^{-1} A_c^T & -X^{-1} \end{pmatrix} < 0 ,$$

where $X = X^T > 0$.
Taking into account the type of matrix A_c, designating $X^{-1} = Y$, get:

$$\begin{pmatrix} -Y & AY + B\Theta Y \\ (AY + B\Theta Y)^T & -Y \end{pmatrix} < 0 .$$

Let us introduce a new variable $Z = \Theta Y$ and write the resulting inequality in the form of linear matrix inequality (14) with respect to the matrices Y and Z.
If the inequality (14) is solvable with respect to unknown matrices Y and Z, then the parameter matrix Θ of the stabilizing regulator by state is found by the formula $\Theta = ZY^{-1}$.
Let us now consider the problem of synthesizing optimal linear-quadratic discrete controllers that ensure the stability of the object and minimize the quadratic functional for a given initial state of the object [8,10,14,15].
Let a linear stationary discrete object is given:

$$\begin{aligned} x_{t+1} &= A_{x_t} + B_{u_t},\ x_0, \\ z_t &= Cx_t + Du_t, \end{aligned} \tag{15}$$

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where $x_t \in \Re^{n_x}$ - object state, $u_t \in \Re^{n_u}$ – controlled input, $z_t \in \Re^{n_z}$ – controlled output, x_0 – initial state, $A \in \Re^{n_x \times n_x}$, $B \in \Re^{n_x \times n_u}$, $C \in \Re^{n_z \times n_x}$ and $D \in \Re^{n_z \times n_u}$ given matrices.

The problem of linear-quadratic control of a given object in the case of a measurable initial state is to find a stabilizing control from the class of linear state feedbacks of the form:

$$u_t = \Theta x_t, \tag{16}$$

minimizing the quadratic functional:

$$J = \sum_{t=0}^{\infty} |z_t|^2. \tag{17}$$

Let's write the quadratic function as:

$$J(\Theta) = \gamma^2 |x_0|^2,$$

where

$$\gamma^2 = \frac{x_0^T P_0 x_0}{|x_0|^2},$$

It depends on the initial conditions of the object x_0. Therefore, according to [5,10], the optimal linear-quadratic control problem can be reformulated as follows: for the given initial conditions of the plant $x_0 \neq 0$ to find:

$$\gamma_* = \min\{\gamma \geq 0 : \exists \Theta \quad J(\Theta) \leq \gamma^2 |x_0|^2\}.$$

Matrix Θ_* and the corresponding γ_*, determine the parameters of the linear-quadratic state control law.

In what follows, instead of this problem, we will consider the problem of suboptimal linear-quadratic control, which consists in the following: for the given initial conditions of the object $x_0 \neq 0$ to find [3,9,10]:

$$\gamma_* = \inf\{\gamma > 0 : \exists \Theta \quad J(\Theta) < \gamma^2 |x_0|^2\}. \tag{18}$$

Suboptimal control parameters matrix Θ_* is determined from the condition:

$$J(\Theta) < (\gamma_*^2 + \varepsilon)|x_0|^2.$$

for whatever small $\varepsilon > 0$. Since, in fact, the parameters of the optimal and suboptimal linear-quadratic control laws are different arbitrarily a little bit, then we will use the term “optimal control” below.

Thus, the posed problem of linear-quadratic control of a discrete object (15) is reformulated as follows: find a stabilizing control from the class of linear state feedbacks of the form (16), which provides at the minimum possible value $\gamma_* > 0$ fulfillment of the inequality:

$$J < \gamma^2 |x_0|^2 \tag{19}$$

for a given initial state of the object $x_0 \neq 0$.

Let us write the equations of the closed-loop system (15), (16):

$$\begin{aligned} x_{t+1} &= A_c x_t, \quad A_c = A + B\Theta, \\ z_t &= C_c x_t, \quad C_c = C + D\Theta. \end{aligned} \tag{20}$$

Next, consider two options:

Option 1. Let a stable discrete object be given:

$$x_{t+1} = A x_t, \quad z_t = C_c x_t \ , \tag{21}$$

with a given initial state $x_0 \neq 0$.

Then the following statements are equivalent:

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a) $J < \gamma^2 |x_0|^2$;

b) $\exists X = X^T > 0$, satisfying inequalities

$$A^T XA - X + C^T C < 0,$$
$$x_0^T X x_0 < \gamma^2 |x_0|^2. \tag{22}$$

We consider the quadratic function $J = \sum_{t=0}^{\infty} |z_t|^2$ and substitute the output equation in the expression for the function z_t, of (21):

$$J = \sum_{t=0}^{\infty} z_t^T z_t = \sum_{t=0}^{\infty} x_t^T C^T C x_t . \tag{23}$$

Since, there is a solution in the first of the equations of the object (21)

$$x_t = A^t x_0 ,$$

where $x_0 \neq 0$ – given initial state, then we substitute this solution into (23) and obtain:

$$J = \sum_{t=0}^{\infty} x_0^T (A^T)^t C^T C A^t x_0 = x_0^T P_0 x_0 < \gamma^2 |x_0|^2,$$

where the matrix $P_0 = \sum_{t=0}^{\infty} (A^T)^t C^T C A^t = P_0^T \geq 0$ is a solution to the Lyapunov equation [2,9]:

$$A^T PA - P + C^T C = 0 . \tag{24}$$

Let us take a matrix $X > P_0$ such that $X = X^T > 0$. X will be the only solution to the equation:

$$A^T XA - X + C^T C + \varepsilon^2 I = 0.$$

This means that the equality,

$$A^T XA - X + C^T C < 0 ,$$

where $X = X(\varepsilon)$. The fulfillment of this equality ensures the asymptotic stability of the discrete object (21).
Since the matrix $X = X(\varepsilon)$ depends on ε continuously, then you can pick up enough less $\varepsilon > 0$ so that

$$x_0^T P_0 x_0 < x_0^T X_0 x_0 < \gamma^2 |x_0|^2, \qquad x_0 \neq 0 ,$$

It is done in paragraph b).

There is a matrix $X = X^T > 0$, satisfying inequalities (22). Hence it follows that object (21) is asymptotically stable and Lyapunov equation (24) will have a unique solution P_0, Where $P_0 < X$, $P_0 = P_0^T \geq 0$. Because:

$$P_0 < X , \ x_0^T X x_0 < \gamma^2 |x_0|^2,$$

then

$$x_0^T P_0 x_0 < x_0^T X_0 x_0 < \gamma^2 |x_0|^2 ,$$

for a given initial state $x_0 \neq 0$.

The fulfillment of these conditions is equivalent to paragraph a) .

Let's apply the option 1 in closed-loop system (20) and present the synthesis of linear-quadratic controllers based on the theory of linear matrix inequalities.

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Variant 2. For the existence of a linear-quadratic state controller of the form (16) for a discrete plant (15), it is necessary and sufficient that there is exist $(n_x \times n_x)$-matrix $Y = Y^T > 0$, $(n_u \times n_x)$-matrix Z and the number $\gamma^2 > 0$, satisfying the linear matrix inequalities [13]:

$$\begin{pmatrix} -Y & 0 & AY+BZ \\ 0 & I & CY+DZ \\ (AY+BZ)^T & (CY+DZ)^T & -Y \end{pmatrix} < 0, \qquad (25)$$

$$\begin{pmatrix} Y & x_0 \\ x_0^T & \gamma^2 |x_0|^2 \end{pmatrix} > 0, \qquad (26)$$

where $x \neq 0$ – given initial state of the object.

If inequalities (25) and (26) are solvable with respect to the matrices Y, Z and numbers $\gamma^2 > 0$, then the parameters of the linear-quadratic controller (16) by state are found as $\Theta = Z_* Y_*^{-1}$, where Y_* и Z_* – matrices corresponding to the minimum possible $\gamma_*^2 > 0$.

According to 2), to fulfill the control goal (19), it is necessary and sufficient that the matrix exists $X = X^T > 0$, satisfying the inequalities:

$$A_c^T X A_c - X + C_c^T C_c < 0, \qquad (27)$$

$$x_0^T X x_0 < \gamma^2 |x_0|^2, \qquad (28)$$

for a given initial condition of the object $x_0 \neq 0$.

If we directly substitute the matrices of the closed-loop system into inequality (27) A_c, and C_c, then we obtain a matrix inequality that is not linear with respect to the unknown matrices $X = X^T > 0$ and Θ. Therefore, it is necessary to represent the resulting inequality in the form of a linear matrix inequality. To do this, we multiply (28) left and right by the matrix $X^{-1} > 0$ and get

$$X^{-1} A_c^T X A_c X^{-1} - X^{-1} + X^{-1} C_c^T C_c X^{-1} < 0,$$

Or,

$$X^{-1} \begin{pmatrix} A_c^T & C_c^T \end{pmatrix} \begin{pmatrix} X & 0 \\ 0 & I \end{pmatrix} \begin{pmatrix} A_c \\ C_c \end{pmatrix} X^{-1} - X^{-1} < 0.$$

Taking into account Schur's lemma, the last inequality is written as follows [11-13]:

$$\begin{pmatrix} -\begin{pmatrix} X & 0 \\ 0 & I \end{pmatrix}^{-1} & \begin{pmatrix} A_c \\ C_c \end{pmatrix} X^{-1} \\ X^{-1} \begin{pmatrix} A_c \\ C_c \end{pmatrix}^T & -X^{-1} \end{pmatrix} < 0,$$

those

$$\begin{pmatrix} -X^{-1} & 0 & A_c X^{-1} \\ 0 & -I & C_c X^{-1} \\ X^{-1} A_c^T & X^{-1} C_c^T & -X^{-1} \end{pmatrix} < 0,$$

where $X = X^T > 0$.

Taking into account the type of matrices A_c, C_c and designating $X^{-1} = Y$, *get*

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$$\begin{pmatrix} -Y & 0 & AY + B\Theta Y \\ 0 & -I & CY + D\Theta Y \\ YA^T + (\Theta Y)^T B^T & YC^T + (\Theta Y)^T D^T & -Y \end{pmatrix} < 0. \quad (29)$$

In (25) we denote $Z = \Theta Y$, we obtain matrix inequality (29) with respect to the matrices Y and Z .
Let us to consider the condition (28) and to represent in the form of a linear matrix inequality with respect to the matrix $Y = Y^T > 0$ and numbers $\gamma^2 > 0$. For this, we can write inequality (28) as below:

$$\begin{pmatrix} X^{-1} & x_0 \\ x_0^T & \gamma^2 |x_0|^2 \end{pmatrix} < 0,$$

which is equivalent to the linear matrix inequality (26), where $X^{-1} = Y$, $x_0 \neq 0$ – given initial state.
If inequalities (25) and (26) are solvable, then the matrix of parameters Θ linear-quadratic controller is found by the formula $Z = \Theta Y^{-1}$.

IV. CONCLUSION

In conclusion, an approach to solving the problem of synthesis of stabilizing state controllers for discrete plants is proposed. It is based on the use of linear matrix inequalities. An attitude to solve the problem of synthesis of linear-quadratic state controllers is developed by using of apparatus of linear matrix inequalities. Vital and sufficient conditions for the existence of linear-quadratic discrete state controllers are formulated.

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