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Amplitude-phase relationship of autoparametric oscillations at fundamental frequency for a "m" - phase symmetrical zero conductor circuit

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ABSTRACT:This paper investigates a multi-phase ferroresonant circuit with zero conductor and no ferromagnetic coupling between the non-linear coils. An equilibrium equation for the ferroresonant circuit is presented. The volt-current characteristic of one phase of a three-phase ferroresonant circuit and the topographic diagram of phase and line voltages of a three-phase asymmetrical ferroresonant circuit (at fundamental harmonic) are determined.

KEY WORDS: Autoparametric oscillations, ferroresonant circuit, volt-current response, symmetrical and asymmetrical circuit.

I.INTRODUCTION

It is well known that in three-phase circuits at certain proportions of applied voltage and circuit parameters ferroresonance phenomena are also observed, which are distinguished by more complex laws of establishment and maintenance of free and forced oscillations in them. This is connected with an increase in the number of non-linear elements and in the number of degrees of freedom of the circuit. It is for this reason, up to now, considered that many issues of ferro resonance have not yet been fully investigated, and as a consequence, not been exploited for practical purposes.

In the following it will be shown that in multiphase circuits (particularly three-phase circuits) with magnetic coupling between phases without neutral conductor and in three-phase circuits without magnetic coupling between phases with neutral conductor the ferroresonance phenomena can be used to build various devices for automation, communication, converters and computer engineering.

II. MATHEMATICAL MODEL

Figure 1 shows a "m" - phase symmetrical circuit, each phase of which is a series ferro-resonant circuit consisting of a resistor $R_q = R$, a linear capacitance Cq = C and a non-linear inductance L(i) identical for all phases (here q = 1, 2 ... m is the sequence number of the phase). The neutral point of circuit 0^1 is connected to the zero point of the multiphase sinusoidal voltage system via the neutral conductor with resistance R_0.

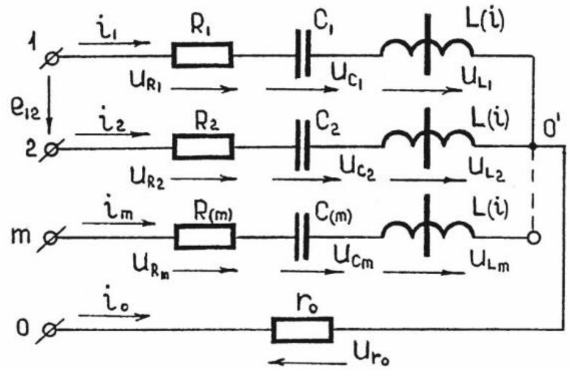

Figure 1. Multiphase ferroresonant circuit with zero conductor and no ferromagnetic coupling between the non-linear coils.

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The equilibrium equations for this chain can be represented as a system

$$
\begin{gathered}
R_1 i_1 \frac{1}{C_1}\int i_1\,dt + \frac{d\varphi_1}{1t} + u_n = u_1 = u_m \sin\omega t,\\
R_2 i_2 \frac{1}{C_2}\int i_2\,dt + \frac{d\varphi_2}{1t} + u_n = u_2 = u_m \sin(\omega t - \frac{2\pi}{m}),\\
R_m i_m \frac{1}{C_m}\int i_{(m)}\,dt + \frac{d\varphi_{(m)}}{1t} + u_n = u_{(m)} = u_m \sin(\omega t + \frac{2\pi}{m}),
\end{gathered}
\tag{1}
$$

here$u_n = R_0 i_0 = R_0\,(i_1 = i_2 + \ldots + i_{(m)})$ - instantaneous voltage value between neutral points 0 and 0^1.
Let in the first approximation the solution for the flux-coupling in the "q" branch be taken as $\Psi q = \Psi_{qm}\sin(\omega t + \beta_q)$, i.e., assuming that the higher harmonics in the coil voltage are absent. Then we can take the approximation in the form $i_L = a\Psi + a\Psi^3$, we can write instead of (1):

$$
\begin{gathered}
R_1(a\varphi_1 + b\varphi_1^3) + \frac{1}{C_1}\int (a\varphi_1 + b\varphi_1^3)dt + \frac{d\varphi_1}{dt} = u_1 - u_n,\\
R_2(a\varphi_2 + b\varphi_2^3) + \frac{1}{C_2}\int (a\varphi_2 + b\varphi_2^3)dt + \frac{d\varphi_2}{dt} = u_2 - u_n,\\
R_m(a\varphi_{(m)} + b\varphi_{(m)}^3) + \frac{1}{C_m}\int (a\varphi_{(m)} + b\varphi_{(m)}^3)dt + \frac{d\varphi_{(m)}}{dt} = u_{(m)} - u_n.
\end{gathered}
\tag{2}
$$

Now, given an approximate solution, we have:

$$
\begin{gathered}
R_1\left[a\varphi_{1m}\sin(\omega t+\beta_1) + \frac{3b}{4}\varphi_{1m}^3\sin(\omega t+\beta_1) - \frac{b}{d4t}\varphi_{1m}^3\sin(3\omega t + 3\beta_1)\right] +\\
+\frac{1}{C_1}\int [a\varphi_{1m}\sin(\omega t + \beta_1) + \frac{3b}{4}\varphi_{1m}^3\sin(\omega t + \beta_1) - \frac{b}{4}\varphi_{1m}^3\sin(3\omega t +\\
+3\beta_1)]dt + \omega\varphi_{1m}\cos(\omega t + \beta_1) = U_m sin\omega t - U_{nm}\sin(\omega t + \gamma_0).
\end{gathered}
\tag{3}
$$

further

$$
\begin{gathered}
R_2\left[a\varphi_{2m}\sin(\omega t+\beta_2) + \frac{3b}{4}\varphi_{2m}^3\sin(\omega t+\beta_2) - \frac{b}{4}\varphi_{2m}^3\sin(3\omega t + 3\beta_2)\right] +\\
+\frac{1}{C_2}\int [a\varphi_{2m}\sin(\omega t + \beta_2) + \frac{3b}{4}\varphi_{2m}^3\sin(\omega t + \beta_2) - \frac{b}{4}\varphi_{2m}^3\sin(3\omega t +\\
+3\beta_2)]dt + \omega\varphi_{2m}\cos(\omega t + \beta_2) = U_m\sin\left(\omega t - \frac{2\pi}{m}\right) - U_{nm}\sin(\omega t + \gamma_0).
\end{gathered}
\tag{4}
$$

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Without getting into the spectral composition of the neutral bias voltage harmonics $U_n(\omega t)$, we remove this term from (3) and (4) by subtracting these equations and convert to a linear source voltage, i.e.

$$R_1\varphi_{1m}(a+\frac{3b}{4}\varphi_{1m}^2)\sin(\omega t+\beta_1)+\left(\omega-\frac{a}{\omega C_1}-\frac{3b}{4\omega C_1}\varphi_{1m}^2\right)\varphi_{1m}\cos(\omega t+$$

$$+\beta_1)-R_2\varphi_{2m}(a+\frac{3b}{4}\varphi_{2m}^2)\sin(\omega t+\beta_2)-\left(\omega-\frac{a}{\omega C_2}-\frac{3b}{4\omega C_2}\varphi_{2m}^2\right)\varphi_{2m}\cos(\omega t+ \quad (5)$$

$$+\beta_2)=U_n 2sin\frac{\pi}{m}\sin\left[\omega t+\frac{(m-2)\pi}{2m}\right]=U_{12m}\sin\left[\omega t+\frac{(m-2)\pi}{2m}\right].$$

The equation (5) contains 4 unknowns Ψ_{1m}, $\Psi_{2m}$and β_1, β_2. In total there will be 2 such unknowns in the system (1), i.e. in each phase the amplitude $\Psi_{qm}$and the initial phase $\beta_q$of the flux in the core of the ferromagnetic element are unknown. We can have "m" equations of (5) type, i.e. for the number of phases of the multiphase circuit.Hence, to solve the whole problem it will be necessary to introduce some phase-amplitude constraints according to the particular condition of the circuit parameters and its operation mode.
For example, let it be required to determine the amplitude-phase relations for the autoparametric oscillations at fundamental frequency in a three-phase unbalanced circuit with zero conductor $R_o\neq 0$ when it is supplied from the source of symmetry of the voltage system (Fig. 2). According to (2) we have a system of exercises

$$R_1(a\varphi_1+b\varphi_1^3)+\frac{1}{C_1}\int(a\varphi_1+b\varphi_1^3)dt+\frac{d\varphi_1}{dt}+U_n=U_m sin\omega t,$$

$$R_2(a\varphi_2+b\varphi_2^3)+\frac{1}{C_2}\int(a\varphi_2+b\varphi_2^3)dt+\frac{d\varphi_2}{dt}+U_n=U_m\sin(\omega t-\frac{2\pi}{3}), \quad (6)$$

$$R_3(a\varphi_3+b\varphi_3^3)+\frac{1}{C_3}\int(a\varphi_3+b\varphi_3^3)dt+\frac{d\varphi_3}{dt}+U_n=U_m\sin(\omega t+\frac{2\pi}{3}).$$

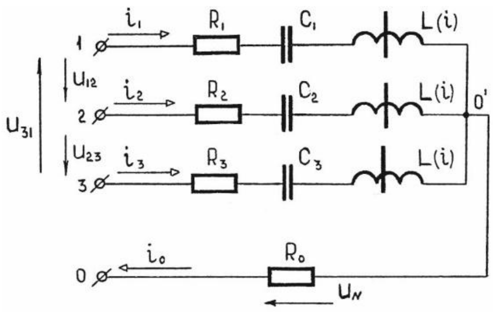

Figure 2. Three-phase ferroresonant circuit with symmetrical power supply.

The fluxes in cores: $\Psi_1(\omega t)$, $\Psi_2(\omega t)$ and $\Psi_3(\omega t)$ will be found in the form $\Psi_1=\Psi_{1m}\sin(\omega t+\beta_1)$, $\Psi_2=\Psi_{2m}\sin(\omega t+\beta_2)$ и $\Psi_3=\Psi_{3m}\sin(\omega t+\beta_3)$. Now according to (5) we can form three equations with five unknowns Ψ_{1m}, Ψ_{2m}, Ψ_{3m}, β_1, β_2 (orβ_3):

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$$R_1\left(a\varphi_{1m}+\frac{3b}{4}\varphi_{1m}^3\right)\sin(\omega t+\beta_1)+\left(\omega\varphi_{1m}-\frac{a}{\omega C_1}\varphi_{1m}-\frac{3b}{4\omega C_1}\varphi_{1m}^3\right)\cos(\omega t+$$
$$+\beta_1)-R_2\left(a\varphi_{2m}+\frac{3b}{4}\varphi_{2m}^3\right)\sin(\omega t+\beta_2)+\left(\omega\varphi_{2m}-\frac{a}{\omega C_2}\varphi_{2m}-\frac{3b}{4\omega C_2}\varphi_{2m}^3\right) \tag{7}$$
$$\cos(\omega t+\beta_2)=\sqrt{3}U\sin\left(\omega t+\frac{\pi}{6}\right),$$

further

$$R_2\left(a\varphi_{2m}+\frac{3b}{4}\varphi_{2m}^3\right)\sin(\omega t+\beta_2)+\left(\omega\varphi_{2m}-\frac{a}{\omega C_2}\varphi_{2m}-\frac{3b}{4\omega C_2}\varphi_{2m}^3\right)\cos(\omega t+$$
$$+\beta_2)-R_3\left(a\varphi_{3m}+\frac{3b}{4}\varphi_{3m}^3\right)\sin(\omega t+\beta_3)+\left(\omega\varphi_{3m}-\frac{a}{\omega C_3}\varphi_{3m}-\frac{3b}{4\omega C_3}\varphi_{3m}^3\right) \tag{8}$$
$$\cos(\omega t+\beta_3)=\sqrt{3}U\sin\left(\omega t-\frac{\pi}{2}\right),$$

further

$$R_3\left(a\varphi_{3m}+\frac{3b}{4}\varphi_{3m}^3\right)\sin(\omega t+\beta_3)+\left(\omega\varphi_{3m}-\frac{a}{\omega C_3}\varphi_{3m}-\frac{3b}{4\omega C_3}\varphi_{3m}^3\right)\cos(\omega t+$$
$$+\beta_3)-R_1\left(a\varphi_{1m}+\frac{3b}{4}\varphi_{1m}^3\right)\sin(\omega t+\beta_1)-\left(\omega\varphi_{1m}-\frac{a}{\omega C_1}\varphi_{1m}-\frac{3b}{4\omega C_1}\varphi_{1m}^3\right) \tag{9}$$
$$\cos(\omega t+\beta_1)=\sqrt{3}U\sin\left(\omega t-\frac{5\pi}{6}\right).$$

If R_1, R_2 and R_3 are sufficiently small, each phase of a three-phase circuit is a ferroresonant (i.e. ferroresonant voltage) branch and therefore has an "N" type volt-current characteristic (Figure 3). The position of the operating point in this characteristic depends on the total phase voltage $U_{ф1}$ (or $U_{ф2}$and $U_{ф3}$). If R1 ≠ R2 ≠ R3 and C1≠ C2≠ C3 the characteristic of the circuit in the individual phases can be either inductive (OA section - fig. 3) or capacitive (ВДsection - there). Theoretically, there are 4 main possible arrangements of operating points on the steady-state voltammetry characteristics of the individual phases:

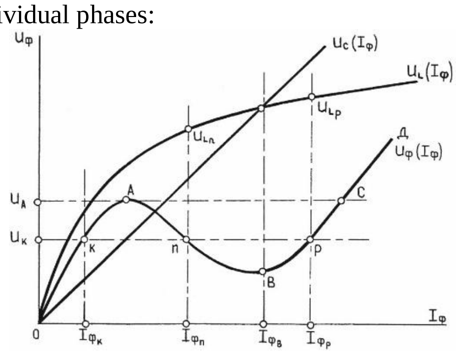

Figure 3. Voltammetry characteristic of one phase of a three-phase ferroresonant circuit.

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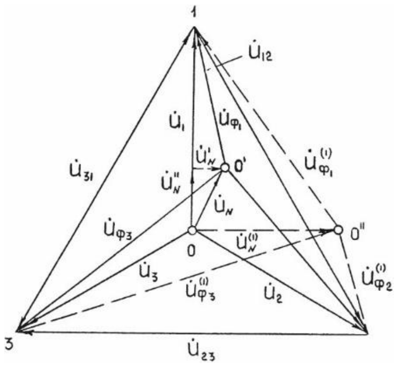

Figure 4. Topographical diagram of the phase and line voltages of a three-phase unbalanced ferroresonant circuit (at fundamental harmonic).

III. CONCLUSION AND FUTURE WORK

The following main conclusions are thus obtained:
1. the nature of all phases is inductive (all operating points on the OA section);
2. the nature of all phases is capacitive (all points are on the БД section);
3 The character of two phases is inductive (OA section) and the third phase is capacitive (БД section);
(4) The character of one phase is inductive and the other two are capacitive.

In the first and second cases the neutral shift 0^1 (at Ro ≠ 0) will occur inside the 1-2-3 triangle of the topographical diagram shown in Fig. 4. In the third and fourth cases, neutral 0^{11} will be outside this triangle. And as we shall show below, this will only be true at $R_o \neq \infty$, and will not be possible at $R_o \neq \infty$, i.e. in the case of a broken neutral conductor.