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GM of AM and HM: A Measure of Central Tendency of Sex Ratio

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ABSTRACT: Four measures of average namely *AGM*, *AHM*, *GHM* and *AGHM* have been developed in some recent studies. It has also found in some other studies that each of these four measures can be regarded as a measure of central tendency of data, in addition to the three commonly used measures namely AM, GM & *HM*. Recently, it has been found that *AHM* can be regarded as a suitable measure of central tendency of data of ratio type. In the current attempt, another measure which is the *GM* of *AM* & *HM* has been identified as equivalent to that of *AHM* as a measure of central tendency of data of ratio type. This paper is based on this measure along with its numerical application in evaluation of central tendency of sex ratio namely male-female ratio and female-male ratio of the states in India.

KEYWORDS: *GM* of *AM* & *HM*, Sex Ratio, Central Tendency, Measure

I. INTRODUCTION

There had already been a lot of research on searching for suitable measure of average [9 , 76]. The first attempt had been made by Pythagoras [12] who developed three measures of average namely Arithmetic Mean (*AM*), Geometric Mean (*GM*) & Harmonic Mean (*HM*) which were later named as Pythagorean means [13 , 38 , 47 , 48 , 58] as a mark of honour to him for his ever-significant discovery. A number of measures of average had been developed consequently in continuation to the three Pythagorean means [36 , 37 , 47 , 48 , 50 , 58]. In the next step of development of measure of average, Kolmogorov [80] formulated one generalized definition of average namely Generalized f - Mean. [72 , 73] from which the existing measures of average can be derived as special cases [36 , 37]. In other studies, Chakrabarty developed two generalized definitions of measure of average namely Generalized f_H – Mean [39] and Generalized f_G – Mean [40 , 42] along with one general method of defining measure of average [47, 48 , 62] as well as the different formulations of average from the first principles [50].

In statistics, the three Pythagorean means are treated/ accepted as three basic measures of central tendency [66 , 65 , 67 , 78 , 79] of numerical data. In fact, if μ is the central tendency of

$$X_1 \ , \ X_2 \ , \ \ldots\ldots\ldots\ldots\ldots \ , \ X_N$$

then they can be described/explained by the model

$$X_i = \mu + \varepsilon_i \quad , \quad (i = 1 , 2 , \ldots\ldots\ldots\ldots , N) \tag{1.1}$$

where

$$\varepsilon_1 , \varepsilon_2 , \ \ldots\ldots\ldots\ldots , \varepsilon_N$$

errors (random in nature) associated to

$$X_1 \ , \ X_2 \ , \ \ldots\ldots\ldots \ , \ X_N$$

respectively [17 – 22 , 27 , 55 , 56 , 63 , 64, 65].

The available statistical methods of estimation of the parameter [1 – 8 , 10, 11 , 15 , 16 , 30, 31 , 32 , 68 , 70 , 71 , 74 , 75 ,77], cannot yield value of the parameter μ accurately [20 , 26 , 28 , 29]. For this reason, therefore, recently some attempts have been made on searching for method(s) of determining the value of parameter μ accurately [18 , 21 , 24 , 25 , 33 , 34 , 38, 43 , 45 , 46 , 51 – 56, 59 , 60]. In these attempts, some methods have been developed for determining such value of parameter. Among these methods, four are based on the measures of average namely Arithmetic-Geometric Mean (*AGM*) [14 , 49, 51 , 57 , 61 , 69], Arithmetic-Harmonic Mean (*AHM*) [52 , 53 , 57 , 59 , 60 , 61], Geometric-Harmonic Mean (*GHM*) [54 , 57, 61] and Arithmetic-Geometric-Harmonic Mean (*AGHM*) [56 , 57, 61]

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respectively. Each of these four has been found to be a measure of parameter μ of the model described by equation (1.1) and consequently a measure of the central tendency [22 , 23 , 55 , 78 , 79] of X_1 , X_2 , …… , X_N , in addition to the usual measures of central tendency namely *AM*, *GM* & *HM*. However, for different types of data different measures are suitable. Recently, it has been found that *AHM* can be regarded as a suitable measure of central tendency of data of ratio type (59 , 60]. In the current attempt, another measure which is the *GM* of *AM* & *HM* has been identified as equivalent to that of *AHM* as a measure of central tendency of data of ratio type. This paper is based on this measure along with numerical application of the measure in estimating the central tendency of sex ratio namely male-female ratio and female-male ratio of the states in India.

II. MEASUTRE OF AVERAGE FROM PYTHAGOREAN MEAN

Let

$$X_1 , X_2 , \ldots\ldots\ldots\ldots\ldots , X_N$$

be N positive numbers or values or observations (not all equal or identical)
and A_0 , G_0 & H_0 the Arithmetic Mean (*AM*), Geometric Mean (*GM*) & Harmonic Mean (*HM*) of them i.e.

$$A_0 = \text{AM}\,(X_1 , X_2 , \ldots\ldots\ldots\ldots , X_N) = (X_1 + X_2 + \ldots\ldots\ldots\ldots\ldots + X_N)/N \quad (2.1)$$

$$G_0 = GM\,(X_1 , X_2 , \ldots\ldots\ldots\ldots , X_N) = (X_1 . X_2 . \ldots\ldots\ldots\ldots . X_N)^{1/N} \quad (2.2)$$

$$\& \quad H_0 = HM\,(X_1 , X_2 , \ldots\ldots\ldots\ldots , X_N) = \{(X_1^{-1} + X_2^{-1} + \ldots\ldots\ldots\ldots\ldots + X_N^{-1})/N \}^{-1} \quad (2.3)$$

which satisfy the inequality namely

$$AM > GM > HM \quad \text{i.e.} \quad H_0 < G_0 < A_0 \quad (2.4)$$

[13 , 38 , 58].

***AGM*:** The *AGM* of X_1 , X_2 , ……… , X_N , is the common point of convergence of the two sequences $\{ A_n \}$ & $\{ G_n \}$ respectively defined by

$$A_{n+1} = AM\,(A_n , G_n) \quad (2.5)$$

$$\& \quad G_{n+1} = GM\,(A_n , G_n) \quad (2.6)$$

with the principal value of the square root [14 , 49, 51 , 57 , 61 , 69].

***AHM*:** The *AHM* of X_1 , X_2 , ……… , X_N , is the common point of convergence of the two sequences $\{ A'_n \}$ & $\{ H'_n \}$ where

$$A'_{n+1} = AM\,(A'_n , H'_n) \quad (2.7)$$

$$\& \quad H'_{n+1} = HM\,(A'_n , H'_n) \quad (2.8)$$

with $A'_0 = A_0$ & $H'_0 = H'_0$ [52 , 53 , 57 , 59 , 60 , 61].

***GHM*:** The *GHM* of X_1 , X_2 , ……… , X_N , is the common point of convergence, denoted by M_{GH} , of the two sequences $\{ G''_n \}$ & $\{ H''_n \}$ where

$$G''_{n+1} = GM\,(G''_n , H''_n) \quad (2.9)$$

$$\& \quad H''_{n+1} = HM\,(G''_n , H''_n) \quad (2.10)$$

with $G''_0 = G_0$ & $H''_0 = H_0$ and the principal value of the square root [54 , 57, 61].

***AGHM*:** The *AGHM* of X_1 , X_2 , ……… , X_N , is the common point of convergence of the three sequences $\{ A'''_n \}$, $\{ G'''_n \}$ & $\{ H'''_n \}$ defined respectively by

$$A'''_{n+1} = AM\,(A'''_n , G'''_n , H'''_n) , \quad (2.11)$$

$$G'''_{n+1} = GM\,(A'''_n , G'''_n , H'''_n) \quad (2.12)$$

$$\& \quad H'''_{n+1} = HM\,(A'''_n , G'''_n , H'''_n) \quad (2.13)$$

where $A'''_0 = A_0$, $G'''_0 = G_0$ & $H'''_0 = H_0$ [56 , 57, 61].

***GM* of AM and *HM*:** The *GM* of *AM* & *HM* of X_1 , X_2 , …………… , X_N , is defined by

$$GM\{ AM\,(X_1 , X_2 , \ldots\ldots\ldots\ldots\ldots , X_N) , HM(X_1 , X_2 , \ldots\ldots\ldots\ldots\ldots , X_N \} = GM\,(A_0 , H_0) \quad (2.14)$$

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III. *GM* OF AM & *HM* AS MEASURE OF CENTRAL TENDENCY OF SEX RATIO

Let

$$x_1 , x_2 , \ldots\ldots , x_N$$

be observed values (which are strictly positive and not all identical) on the Ratio **Male/Female**.
Also let μ be the central tendency of the observed values.
Then x_i can be expressed as

$$x_i = \mu + \varepsilon_i \qquad (3.1)$$

where ε_i is the error associated to x_i for $(i = 1 , 2 , \ldots\ldots , N)$ which is random in nature i.e. each ε_i assumes either positive real value or negative real value with equal probability.
Again since μ is the central tendency of the observed values

$$x_1 , x_2 , \ldots\ldots , x_N$$

therefore, μ^{-1} will be the central tendency of reciprocals

$$x_1^{-1} , x_2^{-1} , \ldots\ldots , x_N^{-1}$$

of the observed values.
Accordingly, the reciprocals can be expressed as

$$x_i^{-1} = \mu^{-1} + \varepsilon_i' \quad , \quad (I = 1 , 2 , \ldots\ldots , N) \qquad (3.2)$$

where

$$\varepsilon_1' , \varepsilon_2' , \ldots\ldots , \varepsilon_N'$$

are the random errors, which assume positive and negative values in random order, associated to are the random errors associated to

$$x_1^{-1} , x_2^{-1} , \ldots\ldots , x_N^{-1}$$

respectively.
Let us now write

$$AM(x_1 , x_2 , \ldots\ldots , x_N) = a_0 \quad \& \quad HM(x_1 , x_2 , \ldots\ldots , x_N) = h_0 \qquad (3.3)$$

and then define the two sequences $\{a'_n\}$ & $\{h'_n\}$ respectively by

$$a'_{n+1} = ½ (a'_n + h'_n) \quad \& \quad h'_{n+1} = \{½(a'^{-1}_n + h'^{-1}_n)\}^{-1} \qquad (3.4)$$

Now since each of a'_n & h'_n is approximate value of μ ,

$a'_0 = \mu + \delta'_0$ & $h'_0 = \mu + e'_0$, for real numbers δ'_0 & e'_0.

This implies, $\delta'_0 > e'_0$ since $a'_0 > h'_0$

By the same logic,

$a'_{n+1} = \mu + \delta'_{n+1}$ & $h'_{n+1} = \mu + e'_{n+1}$, for real numbers δ'_{n+1} & e'_{n+1} .

Since a'_{n+1} is the AM of a'_n & h'_n , therefore, $a'_n > a'_{n+1} > h'_n$
which implies, $\delta'_n > \delta'_{n+1}$ i.e. the sequence $\{\delta'_n\}$ is decreasing.
Moreover, $h_0 - \mu < \delta'_n < a_0 - \mu$ i.e. the sequence $\{\delta'_n\}$ is bounded.
Hence, the sequence $\{\delta'_n\}$ is convergent and converges to a point δ_{AH} in $(h_0 - \mu \ , \ a_0 - \mu)$.
Accordingly, $\{a'_n\}$ & $\{h'_n\}$ and hence $AHM(x_1 , x_2 , \ldots\ldots , x_N)$ converge to the point $\mu + \delta_{AH}$.
Therefore $AHM(x_1 , x_2 , \ldots\ldots , x_N)$, which is the common point of convergence of $\{a'_n\}$ & $\{h'_n\}$, can be a measure of μ and consequently a measure of central tendency of $x_1 , x_2 , \ldots\ldots , x_N$, with deviation δ_{AH} lying within the interval $(h_0 - \mu \ , \ a_0 - \mu)$.

Again,

$a'_1 = ½ (a_0 + h_0)$ & $h'_1 = \{½(a_0^{-1} + h_0^{-1})\}^{-1} = 2 a_0 h_0 /(a_0 + h_0)$

i.e. $(a'_1 h'_1)^{1/2} = (a_0 h_0)^{1/2}$ i.e. $GM(a'_1 , h'_1) = GM(a_0 , h_0)$

Similarly,

$GM(a'_2 , h'_2) = GM(a'_1 , h'_1) = GM(a_0 , h_0)$

In general, it is obtained that

$GM(a'_n , h'_n) = GM(a_0 , h_0)$, for all positive integers n

Since both of $\{a'_n\}$ & $\{h'_n\}$ converge to $AHM(x_1 , x_2 , \ldots\ldots , x_N)$
& $GM(a'_n , h'_n) = GM(a_0 , h_0)$ for all positive integers n
therefore, $GM\{ AHM(x_1 , x_2 , \ldots\ldots , x_N) , AHM(x_1 , x_2 , \ldots\ldots , x_N)\} = GM(a_0 , h_0)$

i.e. $AHM(x_1, x_2, \ldots\ldots, x_N) = GM(a_0, h_0)$

Moreover, it has already been established that $AHM(x_1, x_2, \ldots\ldots, x_N)$ is a measure of central tendency of x_1, x_2, ……….., x_N, with deviation δ_{AH} lying within the interval $(h_0 - \mu, a_0 - \mu)$.

Hence, $GM\{AM(x_1, x_2, \ldots\ldots, x_N), HM(x_1, x_2, \ldots\ldots, x_N)\}$ is a measure of central tendency of x_1, x_2, ……….., x_N, with deviation δ_{AH} lying within the interval $(h_0 - \mu, a_0 - \mu)$.

IV. NUMERICALEXAMPLE

The following table (**Table – 1**) shows the observed values of sex ration in the different states of India in 2011:

Table – 1

State	Value of the Ratio Male/Female	Value of the Ratio Female/Male
Jammu & Kashmir	1.1254138534125111852273651671683	0.88856201384741461016988968870875
Himachal Pradesh	1.0293088804926436613751796256809	0.97152567023553127871119940330966
Punjab	1.11718611741734676457868601138	0.89510600284914783429585712319405
Chandigarh	1.2229968385823537712700642603947	0.81766360177934533455722165869015
Uttarakhand	1.0382445737805593956494862402266	0.96316419584905755859591305415792
Haryana	1.1381499179200197558719403869263	0.878618874592118673847146598073
Delhi	1.1521304409972803426396508480421	0.86795727672502366109786158864161
Rajasthan	1.077386518469311558714857879884	0.92817200035205763708961523638845
Uttar Pradesh	1.0959666766496911194331303675474	0.91243650131493423988837726768373
Bihar	1.0894569681498644304609103396449	0.91788847952225054362107394324387
Sikkim	1.1236943796151050235298618816238	0.88992168879809329247531494722506
Arunachal Pradesh	1.0658345961198241305435082821376	0.93823188292114434272011116216004
Nagaland	1.0742210801874083323111632505218	0.93090707159232088256563955071444
Manipur	1.0150845888535768920299631387912	0.98513957455445833617176866728857
Mizoram	1.0248621894302476437945104610541	0.97574094381990099740878994632108
Tripura	1.0415856043291039214999824955364	0.96007471286444128606000076825568
Meghalaya	1.0113724418785172369610123540989	0.98875543626896326127874988604615
Assam	1.0441048168517855831597956077024	0.95775824788858682201128358123932
West Bengal	1.0526667948213744061675457587868	0.94996821873695430584361430969287
Jharkhand	1.0543346515488809532602154750904	0.9484654597389357492757813425208
Odisha	1.0216767277963741786589610810708	0.97878318336258074151514020087369
Chhattisgarh	1.0094862433659738915914763831542	0.99060289981333128651017560729672
Madhya Pradesh	1.0741921997293521487367677330733	0.93093209972289388478334723747063
Gujarat	1.0878399216924771607664276945985	0.9192528974705997791133158851059
Daman & Diu	1.6170787338884943945947109074086	0.61839907918110990612171575704752
Dadra & Nagar Haveli	1.29217267204182755470193199021	0.77389037985136251032204789430223
Maharashtra	1.0759593940486569345112623151307	0.92940310343605596519523288750508
Andhra Pradesh	1.0072027731513157131279371653056	0.99284873578258743089946488568226
Karnataka	1.0278146308628600560795309711955	0.97293808627776643762353811714322
Goa	1.0274323920462048498411882041409	0.97330005141109938577265470682144
Lakshadweep	1.0565550239234449760765550239234	0.94647223983334842858436735802916
Kerala	0.92224729321594561234305382426448	1.0843078720382305015931455433978
Tamil Nadu	1.0035802105886977594941050244168	0.99643256159206485698216349975338
Pondicherry	0.96391330758747454527714567183158	1.0374376949964980220763382208646
Andaman & Nicobar	1.1415846041303246862866467840864	0.87597537351321775906857066806002
India	1.0607325851848778252519531570732	0.94274467850509882664736426425148

(Source: "Census Report" by Register General of India 2011)

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A. Central Tendency of the Ratio Male/Female:

From the observed values on the ratio **Male/Female** in **Table – 1** it has been obtained that

AM of **Male / Female** = 1.0835068016450523020161865887443

& *HM* of **Male / Female** = 1.0740468088974845410059550737324

Accordingly,

GM of *AM* & *HM* of **Male / Female** = 1.0787664356688097192593273920721

Hence ,

Central Tendency of the **Ratio Male/Female =** 1.0787664356688097192593273920721

B. Central Tendency of the Ratio Female/Male:

From the observed values on **Female/Male** in **Table – 1** it has been obtained that

AM of **Female/Male** = 0.9310581175009550726813265197974

& *HM* of **Female/Male** = 0.92292913942185992242619179784686

Accordingly,

GM of *AM* & *HM* of **Female/Male** = 0.92698471785509679033872230513345

Hence ,

Central Tendency of the **Ratio Female/Male =** 0.92698471785509679033872230513345

IV. DISCUSSION & CONCLUSION

If μ is the central tendency of

$$x_1 , x_2 , \ldots\ldots\ldots , x_N$$

then the central tendency of

$$x_1^{-1} , x_2^{-1} , \ldots\ldots\ldots\ldots , x_N^{-1}$$

should logically be μ^{-1}.

It is seen in the in the above example that the *GM* of *AM* & *HM* of the ratio **Male/Female** is

1.0787664356688097192593273920721

and of the ratio **Female/Male** is

0.92698471785509679033872230513345

These two values are reciprocals each other i.e.

(1.0787664356688097192593273920721)$^{-1}$ = 0.92698471785509679033872230513345

& (0.92698471785509679033872230513345)$^{-1}$ = 1.0787664356688097192593273920721

Thus, *GM* of *AM* & *HM* of can logically be regarded as an acceptable measure of central tendency of data of ratio type.

It is to be noted that the *AHM* of the Ratio **Male/Female** is found to be

1.0787664356688097192593273920721

which is nothing but the *GM* of *AM* & *HM* of the observed values of the Ratio **Male/Female**.

Similarly, the & *AHM* of the Ratio **Male/Female** is found to be

0.92698471785509679033872230513345

which is nothing but the *GM* of *AM* & *HM* of the observed values of the Ratio **Female/Male**.

Thus, the *GM* of *AM* & *HM* and the *AHM* are equivalent yielding identical results. Due to the simplicity in computational work, therefore, the *GM* of *AM* & *HM* is preferable to that of the *AHM* in computational work.

It is, at this stage, to be mentioned that the error of the value of central tendency determined by *AHM* is δ_{AH} which lies in the interval $(h_0 - \mu \ , \ a_0 - \mu)$. Accordingly, the error of the value of central tendency determined by *GM* of *AM* & *HM* is also δ_{AH} lying in the interval $(h_0 - \mu \ , \ a_0 - \mu)$.

Since a_0 & h_0 , being respectively the *AM* & the *GM* of $x_1 , x_2 , \ldots\ldots\ldots , x_N$, depend on the sample size *N* therefore δ_{AH} also depend on the sample size *N*.

Finally, from the meaning of research [35 , 41 , 44] , it can be concluded that the extraction of information on the *GM* of *AM* & *HM* as a measure of central tendency of numerical data of ratio type can be regarded as research findings carrying fundamental importance and high significance in the theory of analysis of data specially of measure of central tendency of data.

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(Dr. Dhritikesh Chakrabarty with his students in his last official working day (December 31, 2001) at Handique Girls' College)

Dr. Dhritikesh Chakrabarty passed B.Sc. (with Honours in Statistics) Examination from Darrang College, Gauhati University, in 1981 securing 1st class &1st position. He passed M.Sc. Examination (in Statistics) from the same university in the year 1983 securing 1st class & 1st position and successively passed M.Sc. Examination (in Mathematics) from the same university in 1987 securing 1st class (5th position). He obtained the degree of Ph.D. (in Statistics) in the year 1993 from Gauhati University. Later on, he obtained the degree of Sangeet Visharad (inVocal Music) in the year 2000 from Bhatkhande Sangeet vidyapith securing 1st class, the degree of Sangeet Visharad (in Tabla) from Pracheen Kala Kendra in 2010 securing 2nd class, the degree of Sangeet Pravakar (in Tabla) from Prayag Sangeet Samiti in 2012 securing 1st class, the degree of Sangeet Bhaskar (in Tabla) from Pracheen Kala Kendra in 2014 securing 1st class and Sangeet Pravakar (in Guitar) from Prayag Sangeet Samiti in 2021 securing 1st class. He obtained Jawaharlal Nehru Award for securing 1st position in Degree Examination in the year 1981. He also obtained Academic Gold Medal of Gauhati University and Prof. V. D. Thawani Academic Award for securing 1st position in Post Graduate Examination in the year 1983.

Dr. Dhritikesh Chakrabarty also did post doctoral research under the Post Doctoral Research Award by the University Grants Commission for the period 2002 – 05.

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Dr. Dhritikesh Chakrabarty served Handique Girls' College, Gauhati University, during the period of 34 years from December 09, 1987 to December 31, 2021, as Professor (first Assistant and then Associate) in the Department of Statistics along with Head of the Department for 9 years and also as Vice Principal of the college. He also served the National Institute of Pharmaceutical Education & Research (NIPER) Guwahati, as guest faculty (teacher cum research guide), during the period from May, 2010 to December, 2016. Moreover, he is a Research Guide (Ph.D. Guide) in the Department of Statistics of Gauhati University and also a Research Guide (Ph.D. Guide) in the Department of Statistics of Assam Down Town University. He has been guiding a number of Ph.D. students in the two universities. He acted as Guest Faculty in the Department of Statistics and also in the Department of Physics of Gauhati University. He also acted as Guest Faculty cum Resource Person in the Ph.D. Course work Programme in the Department of Computer Science and also in the Department of Biotechnology of the same University for the last six years. Dr. Chakrabarty has been working as an independent researcher for the last more than thirty years. He has already been an author of 250 published research items namely research papers, chapter in books / conference proceedings, books etc. He visited U.S.A. in 2007, Canada in 2011, U.K. in 2014 and Taiwan in 2017. He has already completed one post doctoral research project (2002 – 05) and one minor research project (2010 – 11). He is an active life member of the academic cum research organizations namely (1) Assam Science Society (ASS), (2) Assam Statistical Review (ASR), (3) Indian Statistical Association (ISA), (4) Indian Society for Probability & Statistics (ISPS), (5) Forum for Interdisciplinary Mathematics (FIM), (6) Electronics Scientists & Engineers Society (ESES) and (7) International Association of Engineers (IAENG). Moreover, he is a Reviewer/Referee of (1) Journal of Assam Science Society (JASS) & (2) Biometrics & Biostatistics International Journal (BBIJ); a member of the executive committee of Electronic Scientists and Engineers Society (ESES); and a Member of the Editorial Board of (1) Journal of Environmental Science, Computer Science and Engineering & Technology (JECET), (2) Journal of Mathematics and System Science (JMSS) & (3) Partners Universal International Research Journal (PUIRJ). Dr. Chakrabarty acted as members (at various capacities) of the organizing committees of a number of conferences/seminars already held.

Dr. Chakrabarty was awarded with the prestigious SAS Eminent Fellow Membership (SEFM) with membership ID No. SAS/SEFM/132/2022 by Scholars Academic and Scientific Society (SAS Society) on March 27, 2022.